STEADY-STATE / TRANSIENT SIMULATIONS AND CONTROL OF SINGLE-SPOOL JET ENGINES

by

Yeshayahou Levy and Michael Lichtsinder

1.Steady-State / Dynamic

Mathematical Model

2. MATLAB Program DYNJET-D1:

Steady-State Calculations

3. SIMULINK Program DYNJET-D3:

Transient Response Simulations

4. Feed-Back Nonlinear Control for the Single-Spool Engine

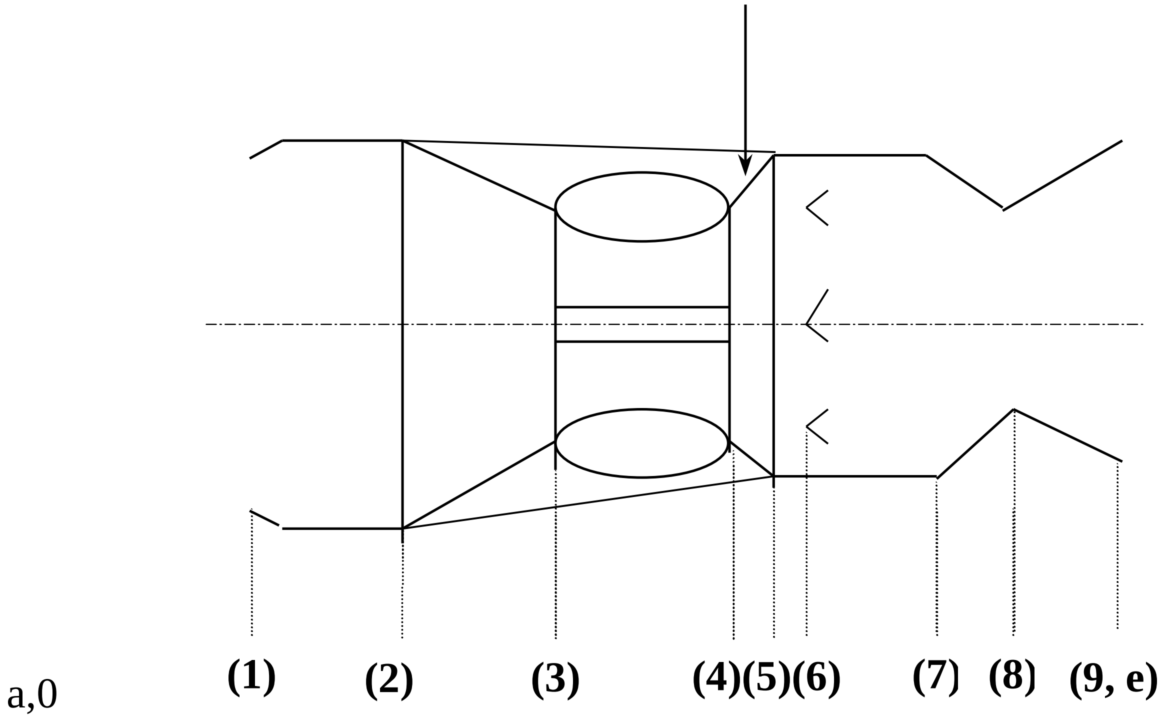
4.1. The Control System Specifications

4.2. Nonlinear Controller

4.3. SIMULINK Program DYNJET-D4:

4.4. Simulations of the Single-Spool Engine Control

ENGINE STATIONS



MATHEMATICAL MODEL

OF SINGLE-SPOOL ENGINE

$$\dot{m}_{2,corr} = \frac{\dot{m}_{a,c}\sqrt{\frac{T_{02}}{T_{ref}}}}{\frac{P_{02}}{P_{ref}}}$$

$$N_{2,corr} = \frac{N}{\sqrt{\frac{T_{02}}{T_{ref}}}}$$

$$\dot{m}_{4,corr} = \frac{\dot{m}_g \cdot \sqrt{T_{04}}}{P_{04}}$$

$$N_{4,corr} = \frac{N}{\sqrt{T_{04}}}$$

Aircraft Intake (0:1) and Engine Intake (1:2):

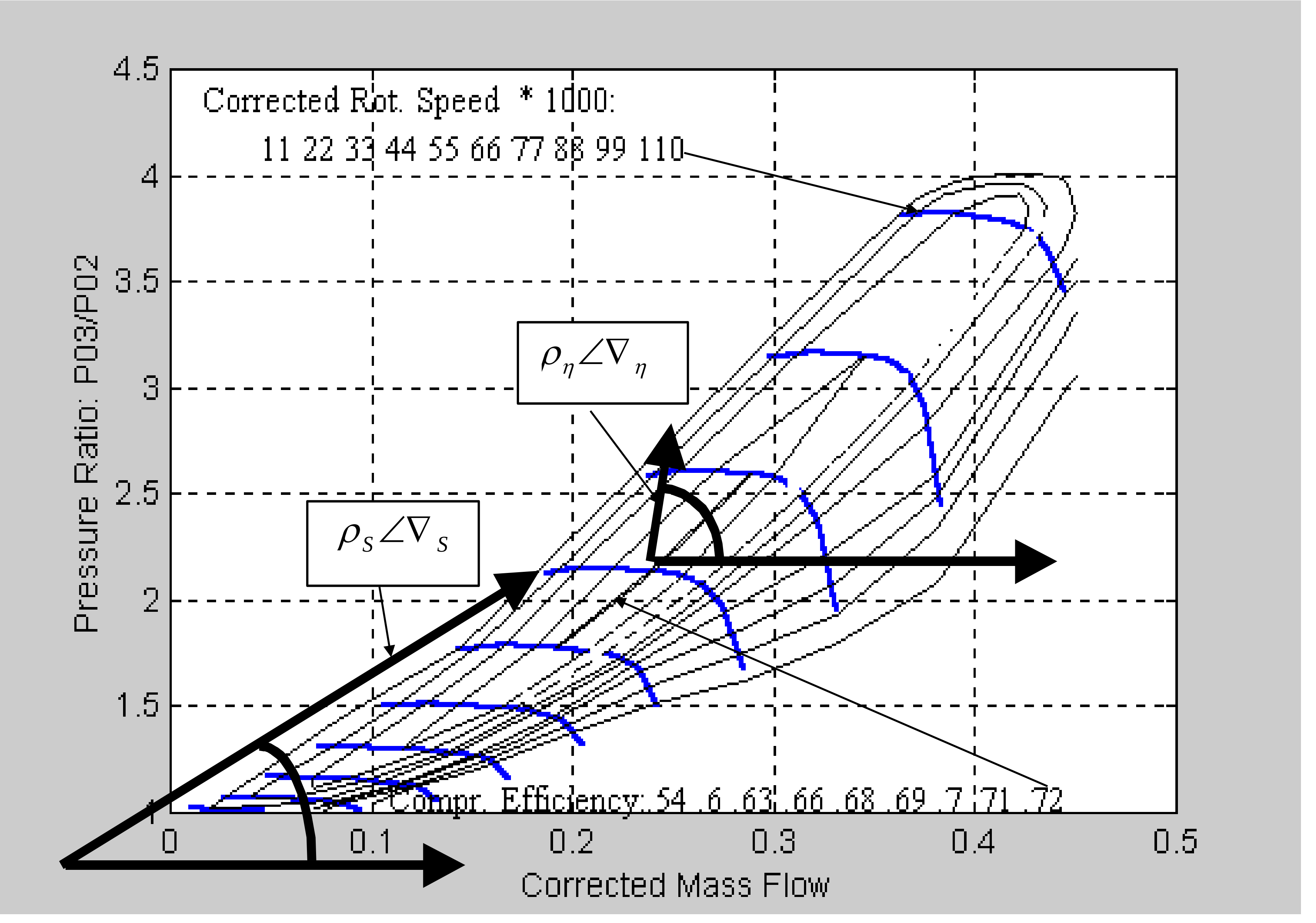
$$\left(\frac{P_{02}}{P_a}\right) = \left[1 + \eta_d \frac{\gamma_a - 1}{2} M^2\right]^{\frac{\gamma_a}{\gamma_a - 1}}$$

$$\left(\frac{T_{02}}{T_a}\right) = 1 + \frac{\gamma_a - 1}{2} M^2$$

Compressor:

$$\left(\frac{P_{03}}{P_{02}}\right) \text{vs. } \dot{\mathrm{m}}_{2,\mathrm{corr}}; \quad \mathrm{N}_{2,\mathrm{corr}} - \textit{parameter}$$

$$\left(\frac{P_{03}}{P_{02}}\right) \text{vs. } \dot{\mathrm{m}}_{2,\mathrm{corr}}; \quad \eta_c - \textit{parameter}$$



The Polar Coordinates for Compressor Map Description

Combustor:

$\left(\frac{P_{04}}{P_{03}}\right)$; for example: $\left(\frac{P_{04}}{P_{03}}\right) = 0.97$

η_b ; for example: $\eta_b = 0.97$

Turbine:

$\dot{m}_{4,corr}$ vs. $\left(\frac{P_{04}}{P_{05}}\right)$, $N_{4,corr}$ = parameter

η_t vs. $\left(\frac{P_{04}}{P_{05}}\right)$, $N_{4,corr}$ = parameter

Exhaust Diffuser and Jet Pipe:

$$\frac{P_{05} - P_{07}}{P_{05}}$$

for example: $$\frac{\mathrm{P}_{05} - \mathrm{P}_{07}}{\mathrm{P}_{05}} = 0.020$$;

Nozzle:

$$\dot{m}_{7,corr} = \dot{m}_{7,corr,theor} \cdot C_D$$

$$\dot{m}_{7,corr,theor} = \sqrt{2 \cdot \eta_n \cdot C_p \cdot \left[1 - \left(\frac{1}{\left(\frac{P_{07}}{P_a}\right)}\right)^{\frac{\gamma_g - 1}{\gamma_g}}\right]} \cdot \frac{A_e}{R} \cdot \frac{1}{\left(\frac{P_{07}}{P_a}\right)} \cdot \frac{1}{1 - \eta_n \left[1 - \left(\frac{1}{\left(\frac{P_{07}}{P_a}\right)}\right)^{\frac{\gamma_g - 1}{\gamma_g}}\right]}$$

Fuel Pump:

$$\tau_f \cdot \frac{d\dot{m}_f}{dt} + \dot{m}_f = (0.0098 \cdot V_f^2 + 0.5095 V_f + 0.8868) \cdot 10^{-3}, \left[\frac{kg}{s}\right]$$

$$\tau_f = \frac{0.2}{1 + 0.204 \cdot V_f}$$

Alternator:

Alternator power;

for example altpower =0… 800, [Watt]

The Engine Equations:

$$f = Cp_{gt} \cdot (T_{04} - T_{03}) / Q_R \cdot \eta_b - Cp_{gt} \cdot T_{04})$$

$$\dot{m}_g = \dot{m}_a + \dot{m}_f$$

$$Cp_{gt} \cdot \dot{m}_g \cdot \left(T_{04} - T_{05}\right) \cdot \eta_m - Cp_{ac} \cdot \dot{m}_a \cdot \left(T_{03} - T_{02}\right) - \mathrm{altpower} = \frac{dE}{dt}$$

$$E = \frac{J \cdot N^2 \cdot \pi^2}{1800}$$

For $\frac{P_{07}}{P_a} < \frac{P_{07}}{P_{cr}}$

$$u_e = \sqrt{2\eta_n \cdot Cp_{gt} \cdot T_{07} \cdot \left(1 - \frac{P_a}{P_{07}}\right)^{\frac{\gamma_g - 1}{\gamma_g}}}$$

$$T = \dot{m}_a \cdot \left[C_x (1+f) u_e - u_a \cdot (1+\varepsilon)\right]$$

For

$$\frac{P_{07}}{P_a} \geq \frac{P_{07}}{P_{cr}}$$

$$ue = \sqrt{2 \cdot \eta_n \cdot Cp_{gt} \cdot T_{07} \cdot \left(1 - \left(\frac{P_{cr}}{P_{07}}\right)^{\frac{\gamma_g - 1}{\gamma_g}}\right)}$$

$$T = \dot{m}_a \left[C_x (1 + f) u_e - u_a \cdot (1 + \varepsilon)\right] + (P_{cr} - P_a) \cdot A$$

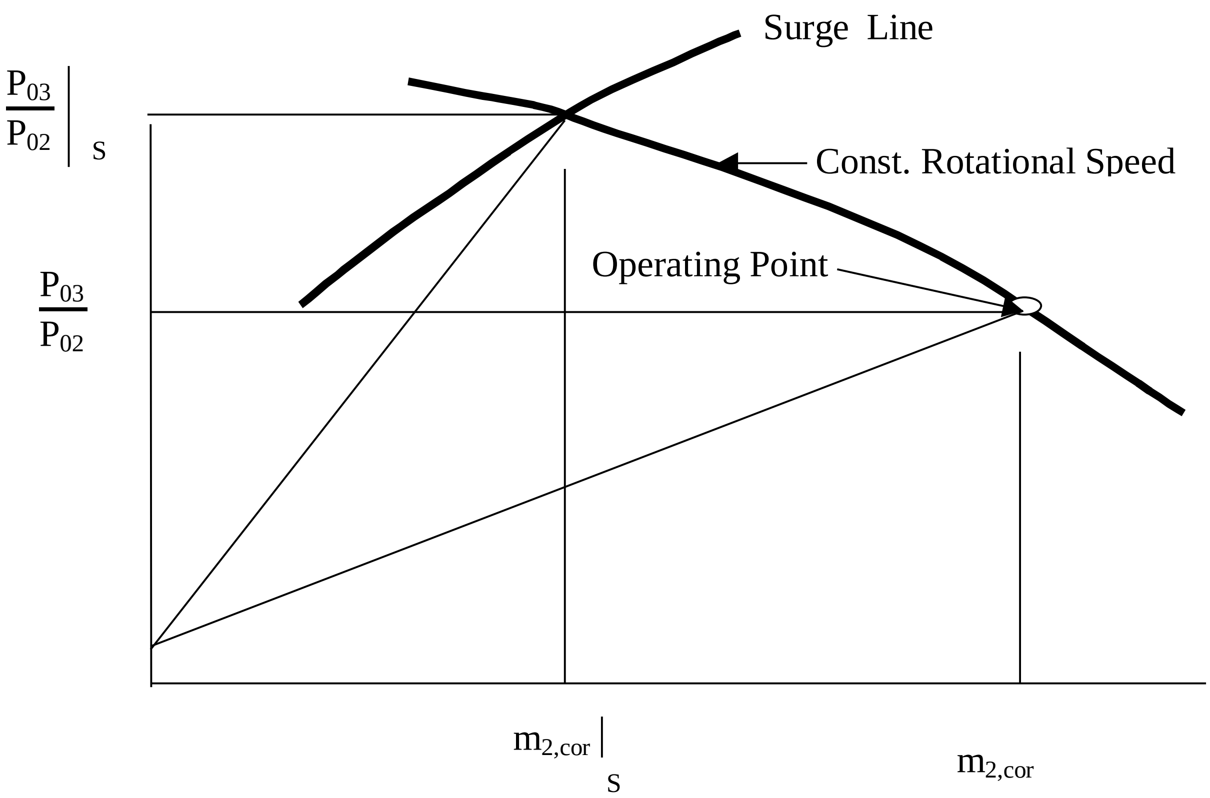
$$SFC = \frac{f \cdot \dot{m}_a}{T}$$

Equivalence Ratio:

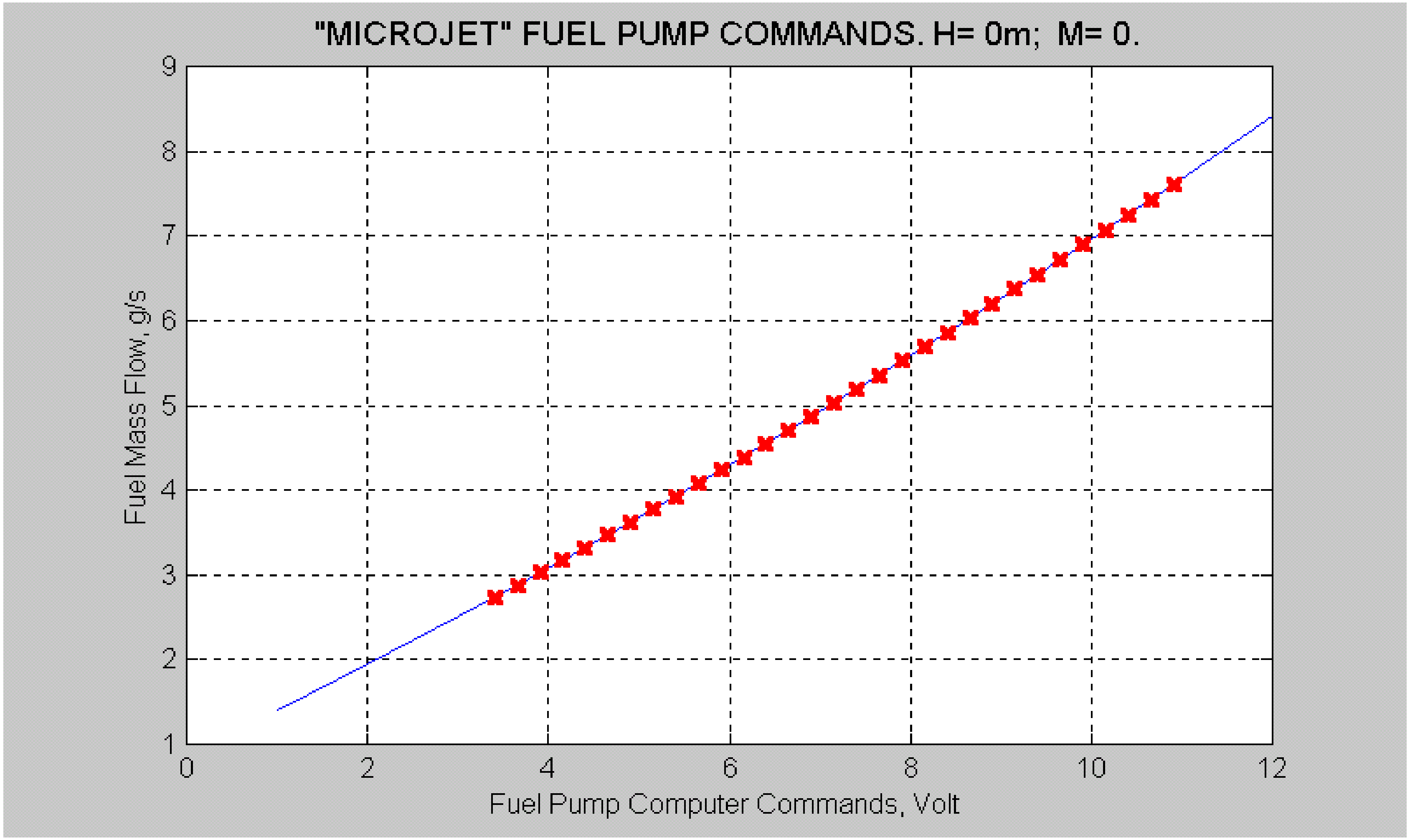
$$\phi = \frac{\dfrac{\dot{m}_f}{\dot{m}_a}}{\left.\dfrac{\dot{m}_f}{m_a}\right|_{stoch}}$$

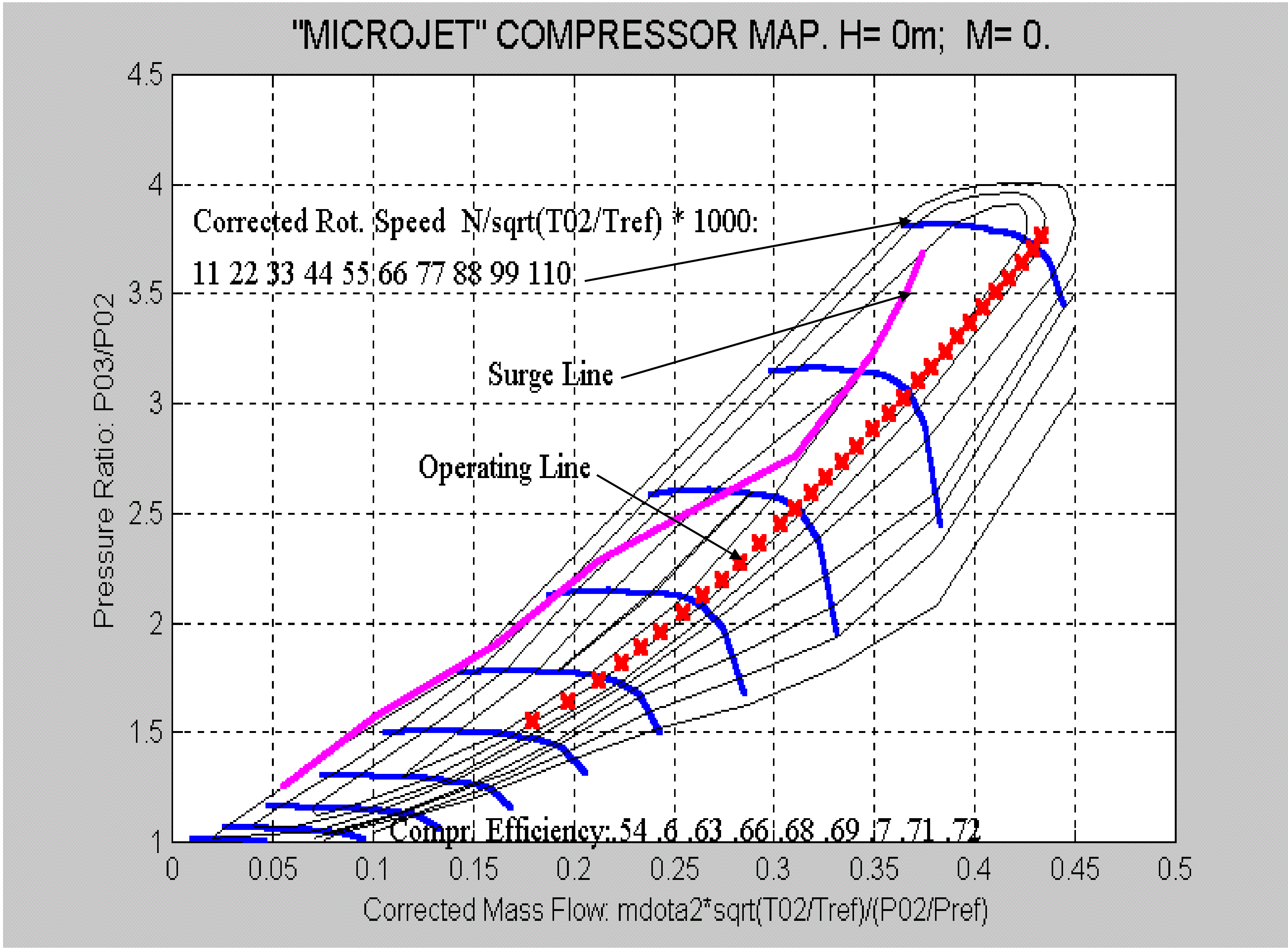
Stall Margin Definition

$$SM = \frac{\dfrac{\left(\dfrac{P_{03}}{P_{02}}\right)_S}{(\dot{m}_{2,cor})_S} - \dfrac{\dfrac{P_{03}}{P_{02}}}{\dot{m}_{2,cor}}}{\dfrac{\dfrac{P_{03}}{P_{02}}}{\dot{m}_{2,cor}}}$$

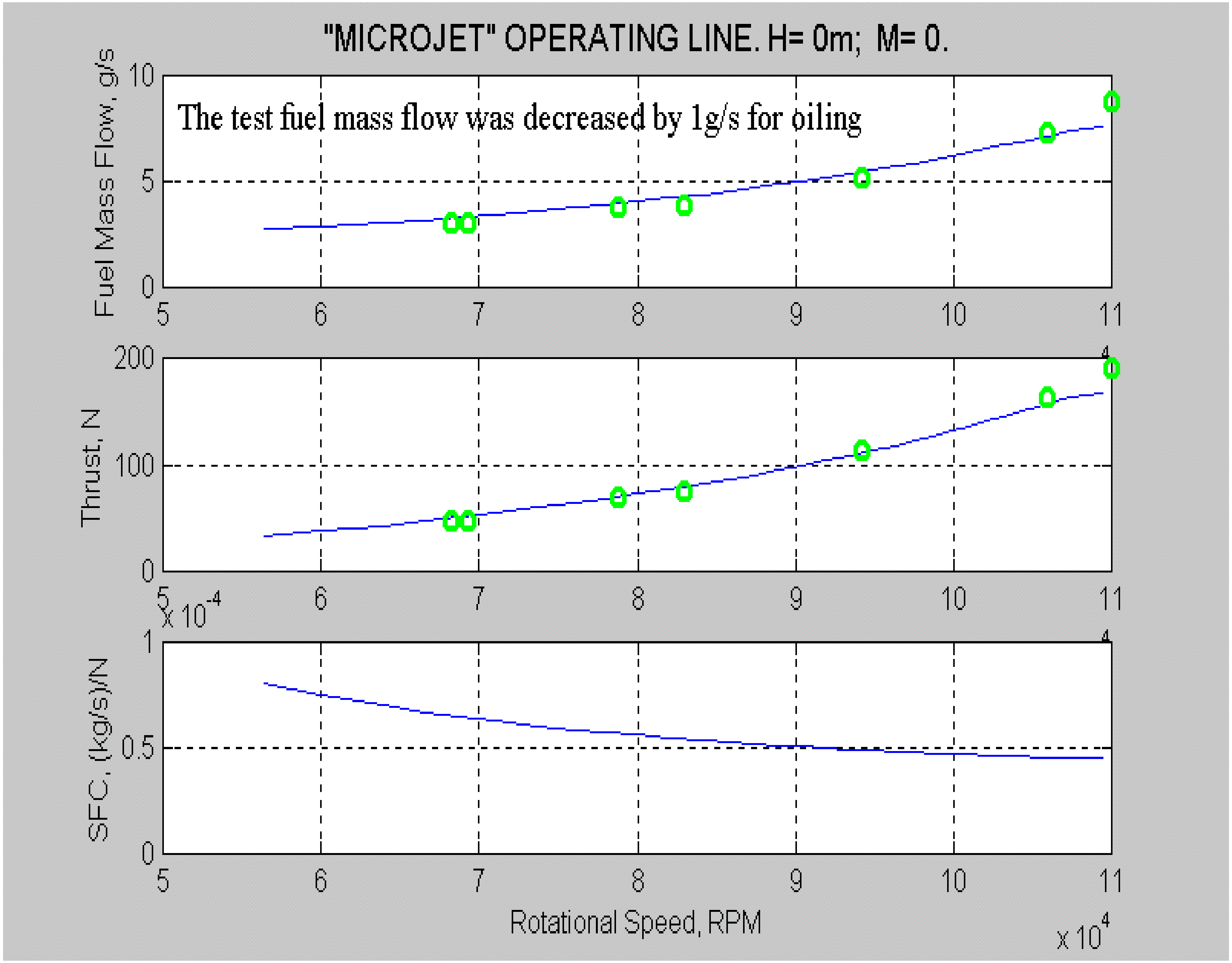


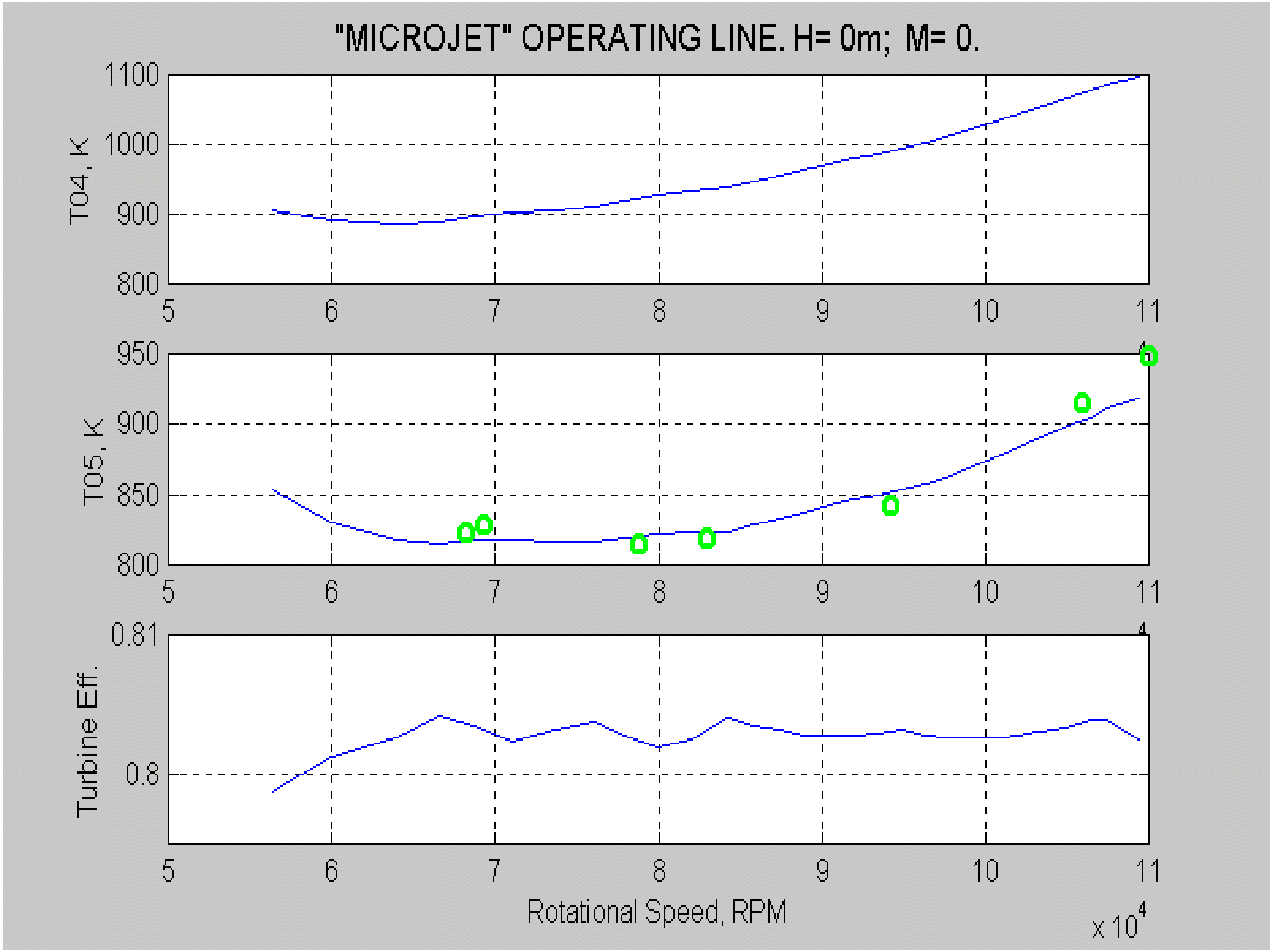
“MICROJET” OPEN LOOP STEADY-STATE SIMULATIONS in MATLAB



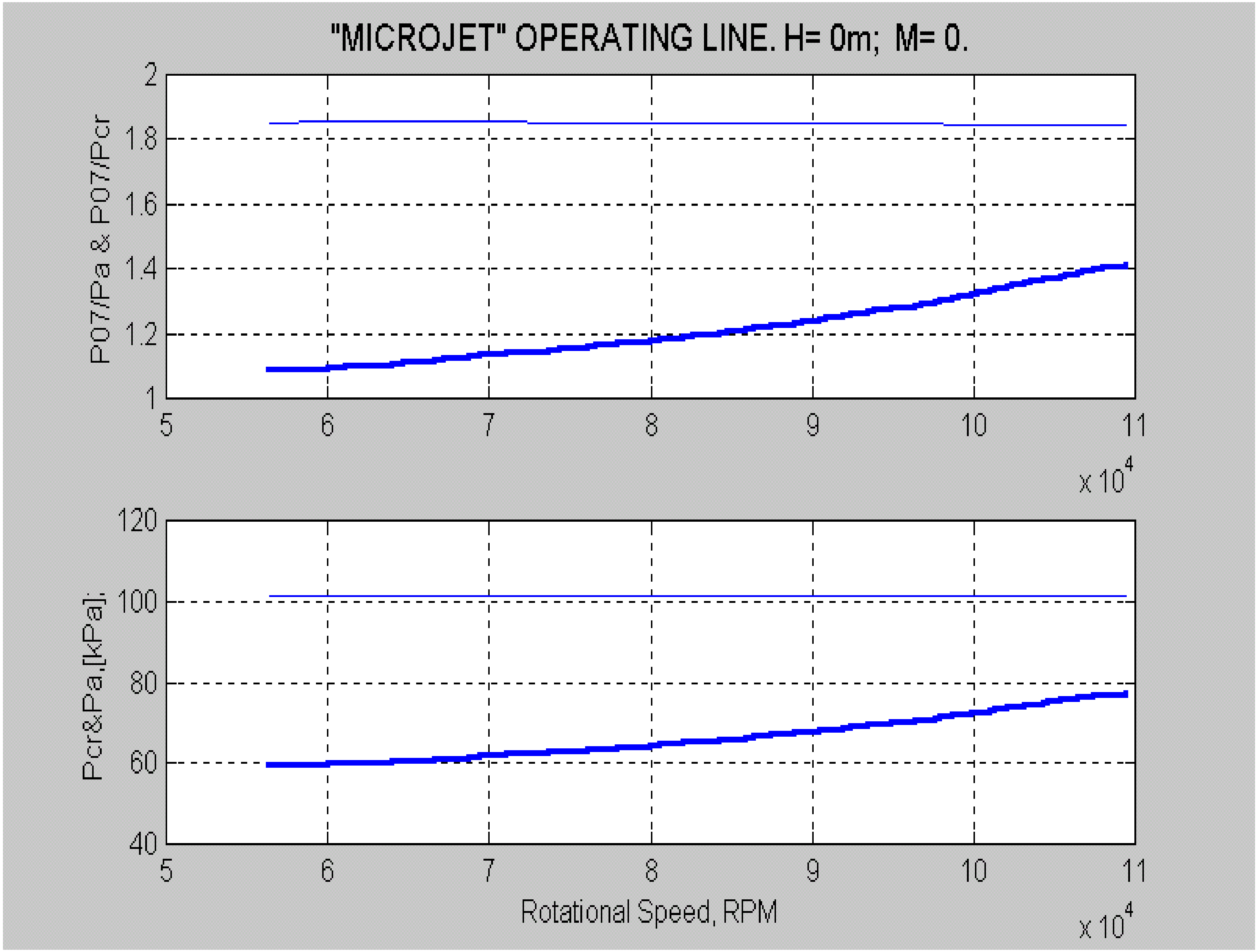


"MICROJET" COMPRESSOR MAP. H= 0m; M= 0.
Corrected Rot. Speed N/sqrt(T02/Tref) * 1000:
11 22 33 44 55 66 77 88 99 110
Surge Line
Operating Line
Compr. Efficiency:.54 .6 .63 .66 .68 .69 .7 .71 .72
Pressure Ratio: P03/P02
Corrected Mass Flow: mdota2*sqrt(T02/Tref)/(P02/Pref)
4.5
4
3.5
3
2.5
2
1.5
1
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5


"MICROJET" OPERATING LINE. H= 0m; M= 0.
The test fuel mass flow was decreased by 1g/s for oiling
Fuel Mass Flow, g/s
Thrust, N
SFC, (kg/s)/N
x 10^-4
Rotational Speed, RPM
x 10^4



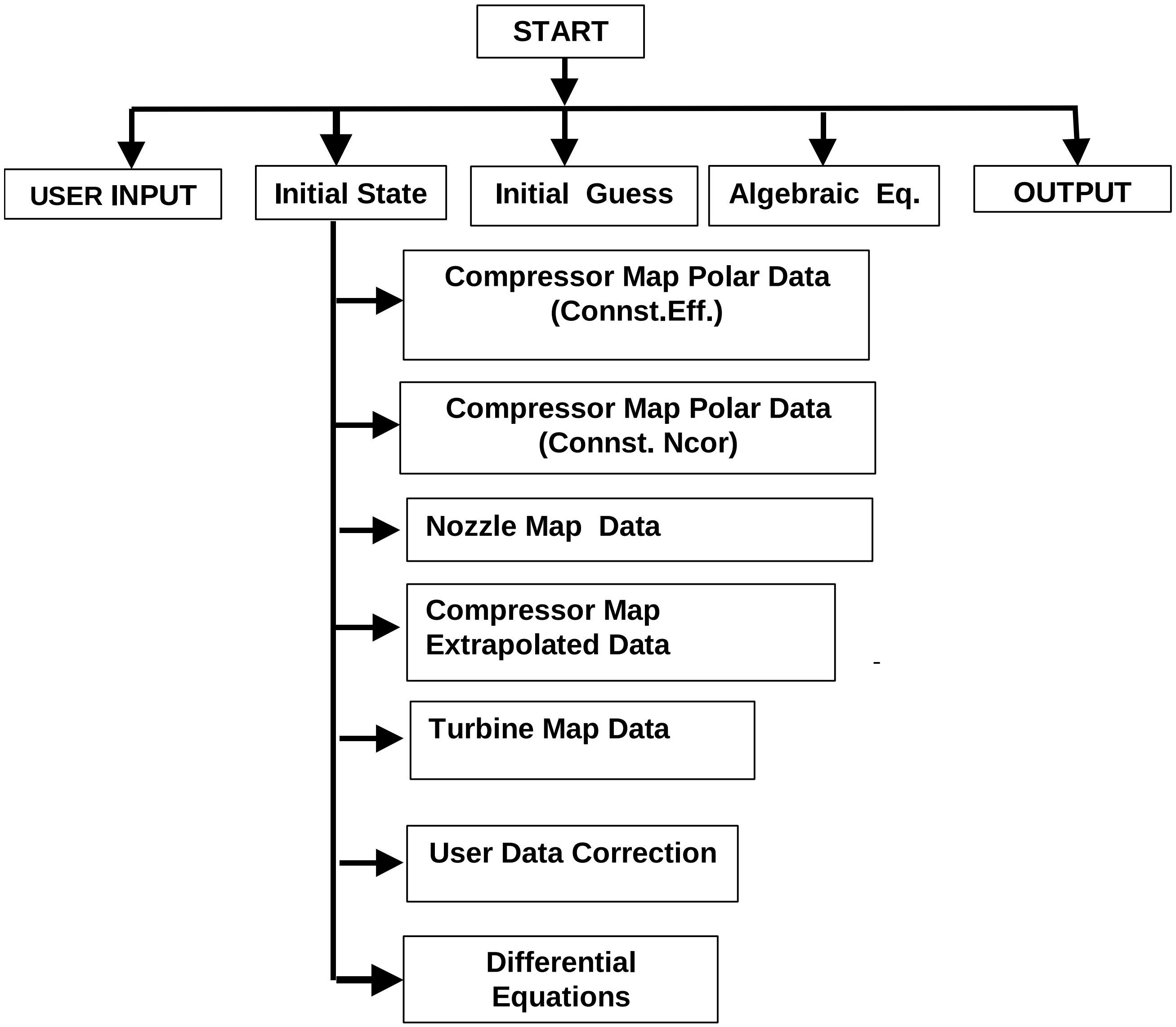
"MICROJET" OPERATING LINE. H= 0m; M= 0.
T04, K
1100
1000
900
800
T05, K
950
900
850
800
Turbine Eff.
0.81
0.8
5
6
7
8
9
10
11
Rotational Speed, RPM
x 10^4


"MICROJET" OPERATING LINE. H= 0m; M= 0.
P07/Pa & P07/Pcr
2
1.8
1.6
1.4
1.2
1
5
6
7
8
9
10
11
x 10^4
Pcr&Pa,[kPa];
120
100
80
60
40
Rotational Speed, RPM
x 10^4

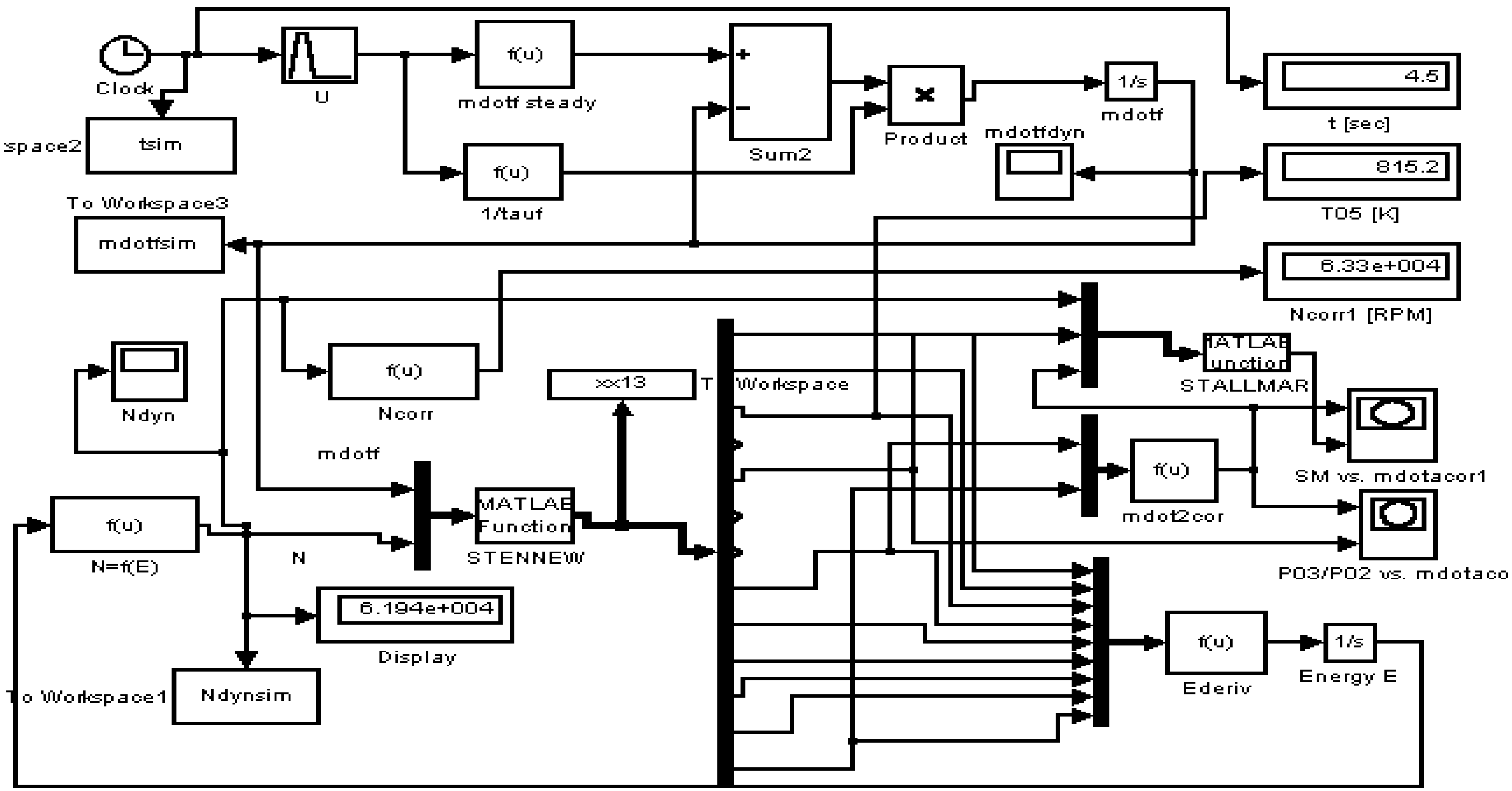
Dynamic Model Input Parametrs

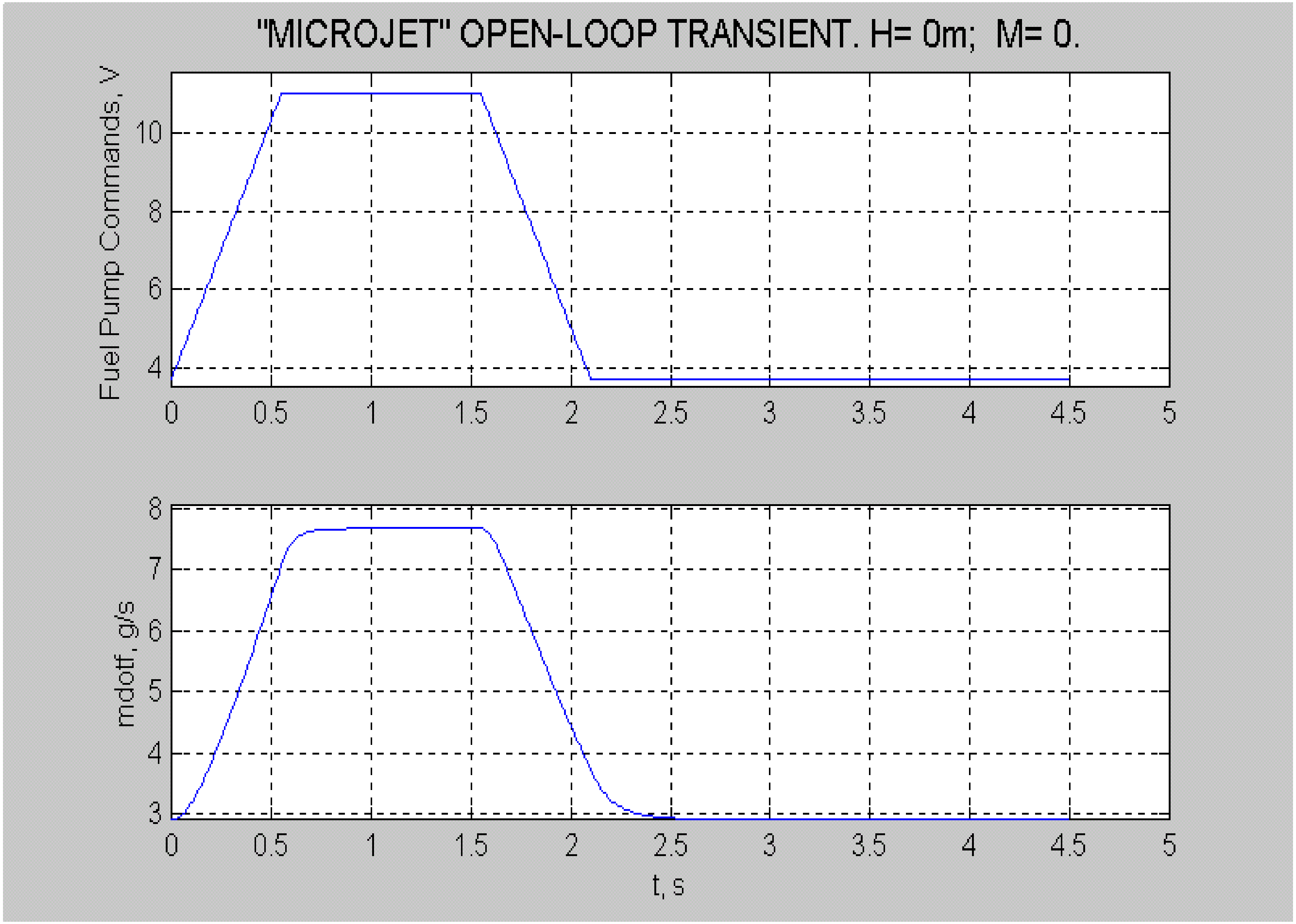
No.	Parameter	Notation
1.	Mach Number	M
2.	Altitude	H, [ft]
3.	Commands:	
	- time vector	t, [s]
	- fuel pump voltage vector	V, [Volt]
4.	Alternator electrical power	W, [Watt]
5.	Stall margin limit	$SM_{\lim}$
6.	Equivalence ratio:	
	- minimum	$\varphi_{\min}$
	- maximum	$\varphi_{\max}$
7.	Turbine inlet temperature limit	$T_{04,\max}$, [K]

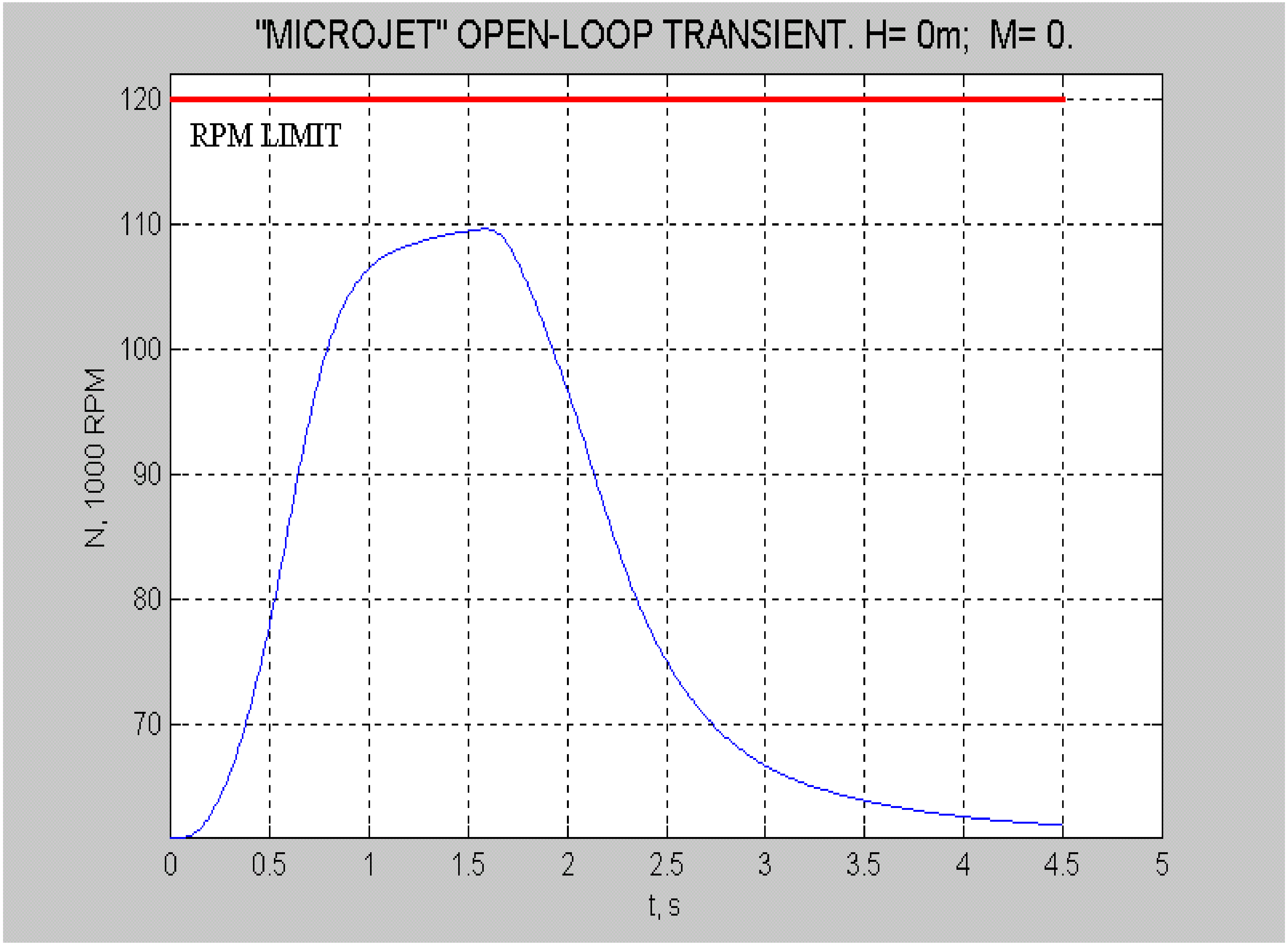
DYNJET-2 BLOCK-DIAGRAM

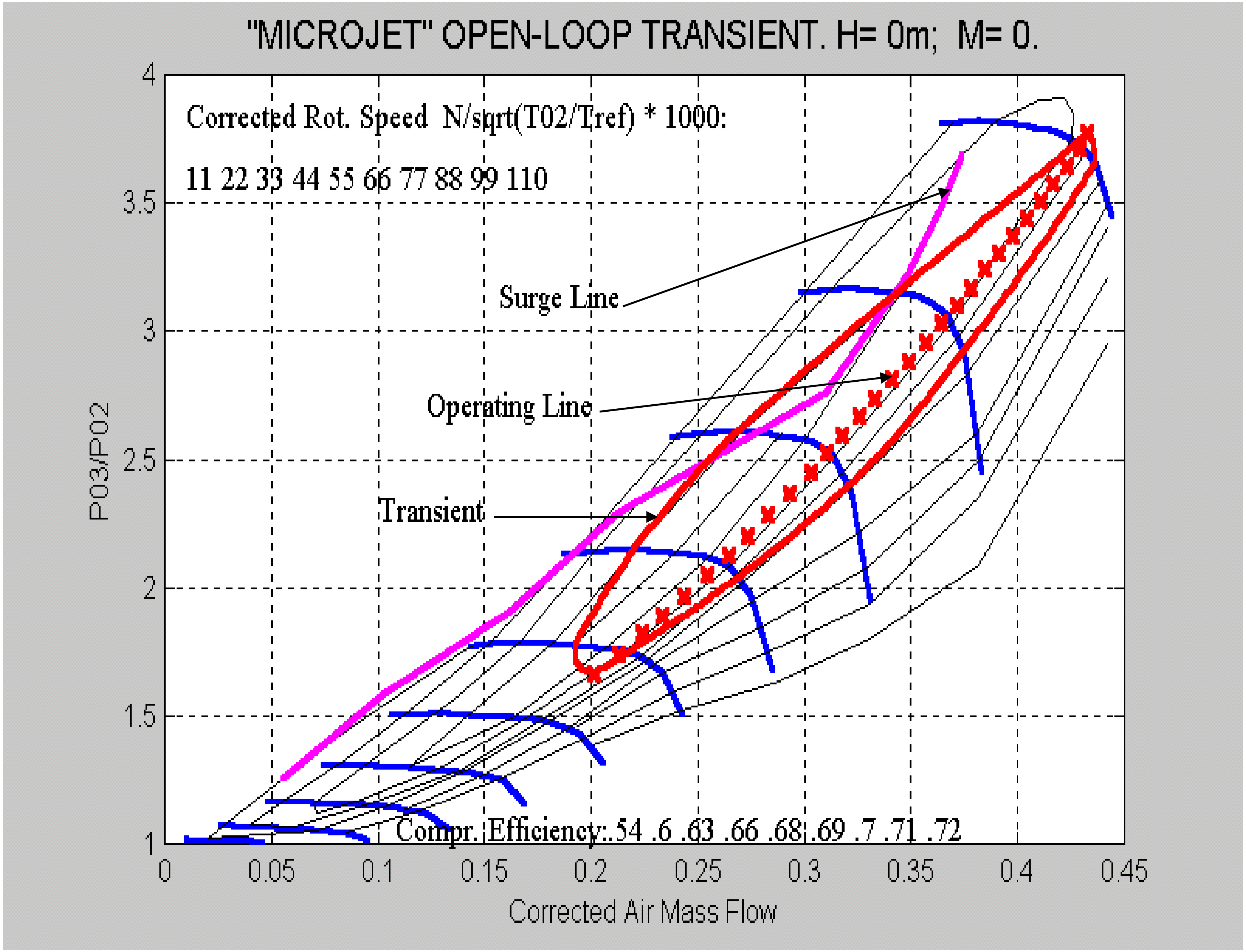


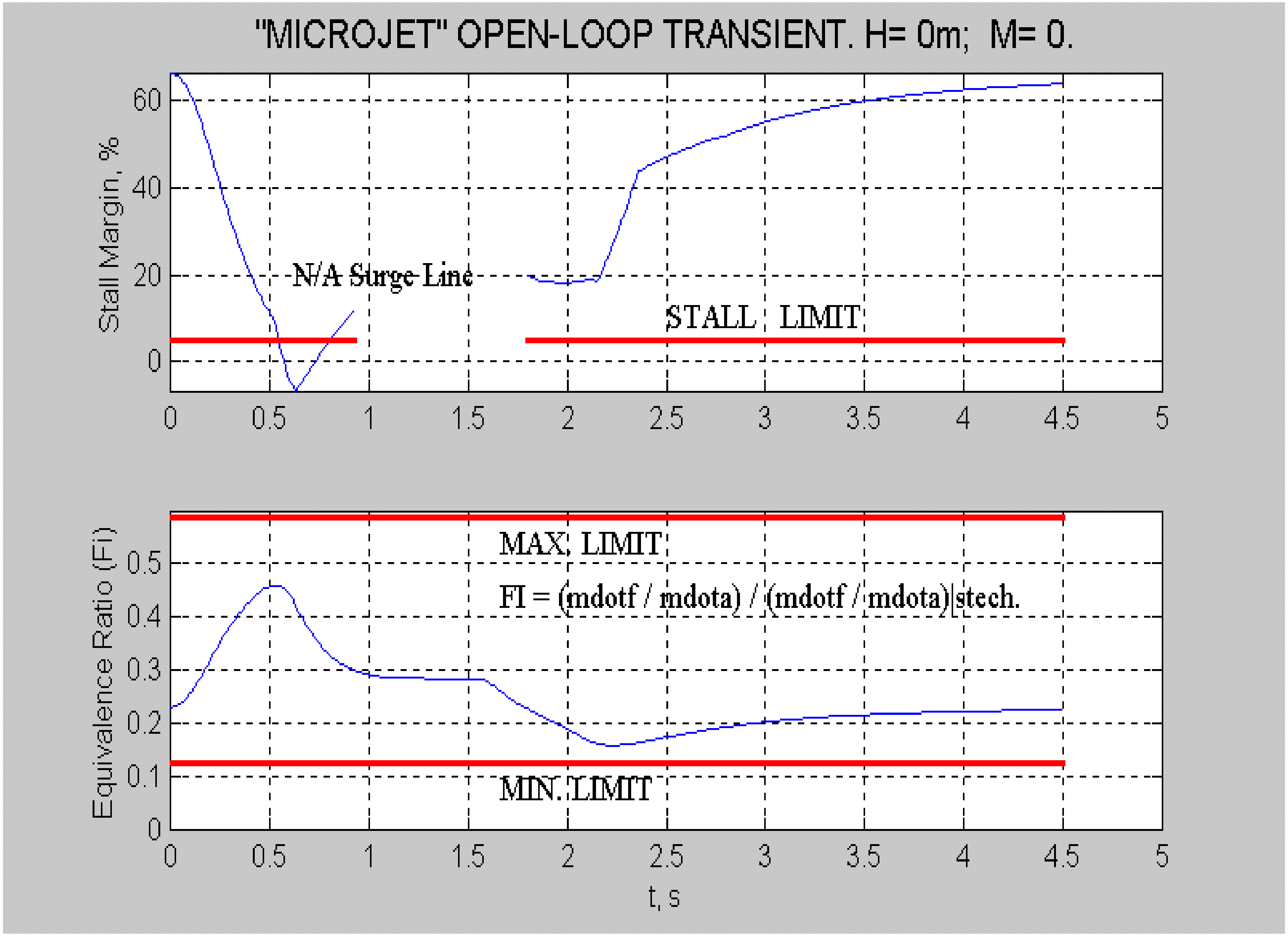
SIMULINK PROGRAM FOR “MICROJET” OPEN-LOOP SIMULATIONS

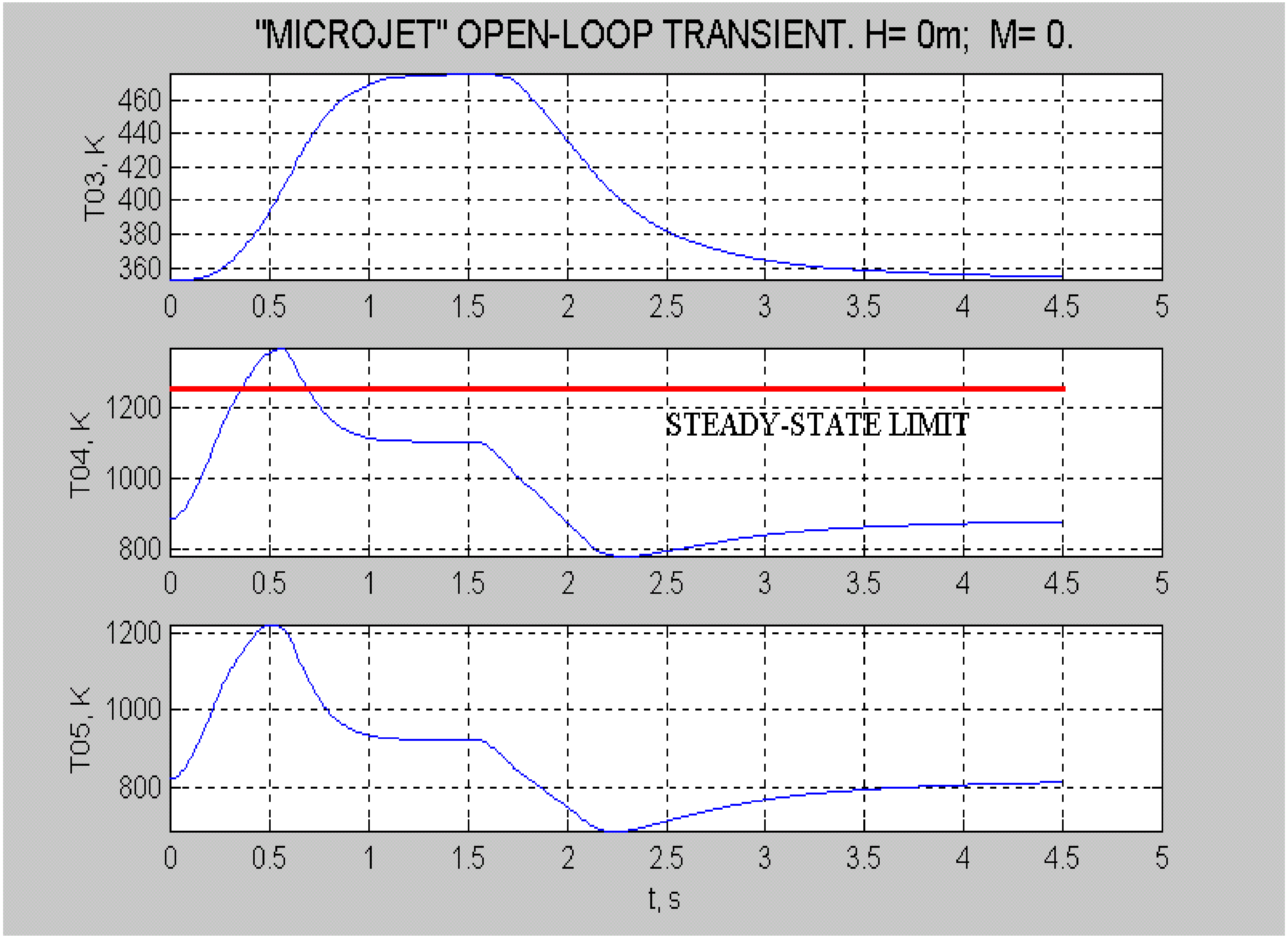



"MICROJET" OPEN-LOOP TRANSIENT. H= 0m; M= 0.
Fuel Pump Commands, V
4
6
8
10
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
mdotf, g/s
3
4
5
6
7
8
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
t, s


"MICROJET" OPEN-LOOP TRANSIENT. H= 0m; M= 0.
RPM LIMIT
N, 1000 RPM
120
110
100
90
80
70
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
t, s


"MICROJET" OPEN-LOOP TRANSIENT. H= 0m; M= 0.
Corrected Rot. Speed N/sqrt(T02/Tref) * 1000:
11 22 33 44 55 66 77 88 99 110
Surge Line
Operating Line
Transient
Compr. Efficiency:.54 .6 .63 .66 .68 .69 .7 .71 .72
P03/P02
Corrected Air Mass Flow
4
3.5
3
2.5
2
1.5
1
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45


"MICROJET" OPEN-LOOP TRANSIENT. H= 0m; M= 0.
Stall Margin, %
60
40
20
0
N/A Surge Line
STALL LIMIT
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
Equivalence Ratio (Fi)
0.5
0.4
0.3
0.2
0.1
0
MAX. LIMIT
FI = (mdotf / mdota) / (mdotf / mdota)|stech.
MIN. LIMIT
t, s


"MICROJET" OPEN-LOOP TRANSIENT. H= 0m; M= 0.
T03, K
460
440
420
400
380
360
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
T04, K
1200
1000
800
STEADY-STATE LIMIT
T05, K
1200
1000
800
t, s

CONTROL PROBLEM:

1. Corrected rotational speed (Ncor) control:

a) Steady-state error: <1%

b) Settling time: minimum

c) Overshoot: <2%

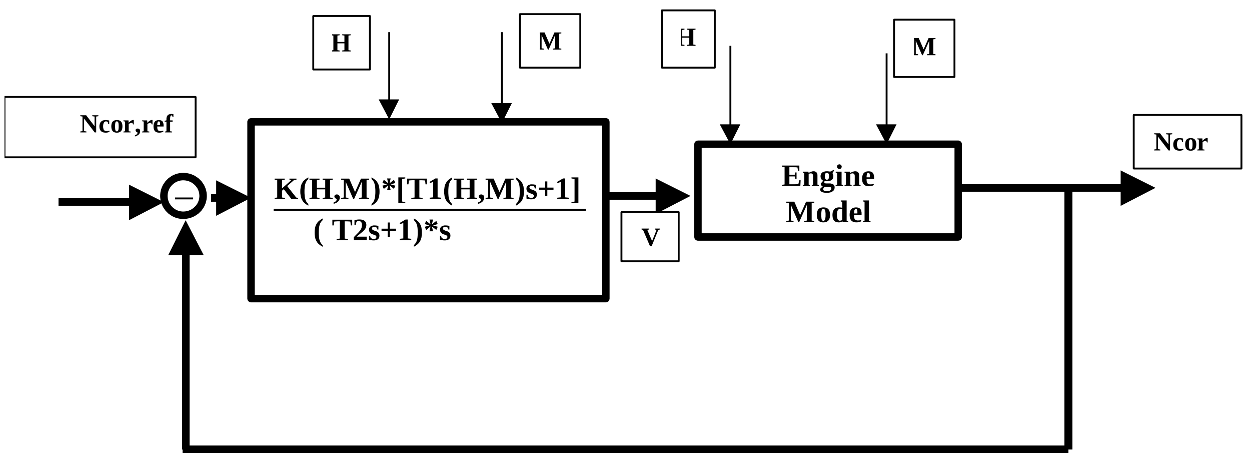
2. Stall margin: >5%

3. T05: <T05max

4. Equivalence Ratio: $\phi_{min} < \phi < \phi_{max}$

Engine Control System

Flight Conditions: 0<H<20,000ft and 0<M<0.7



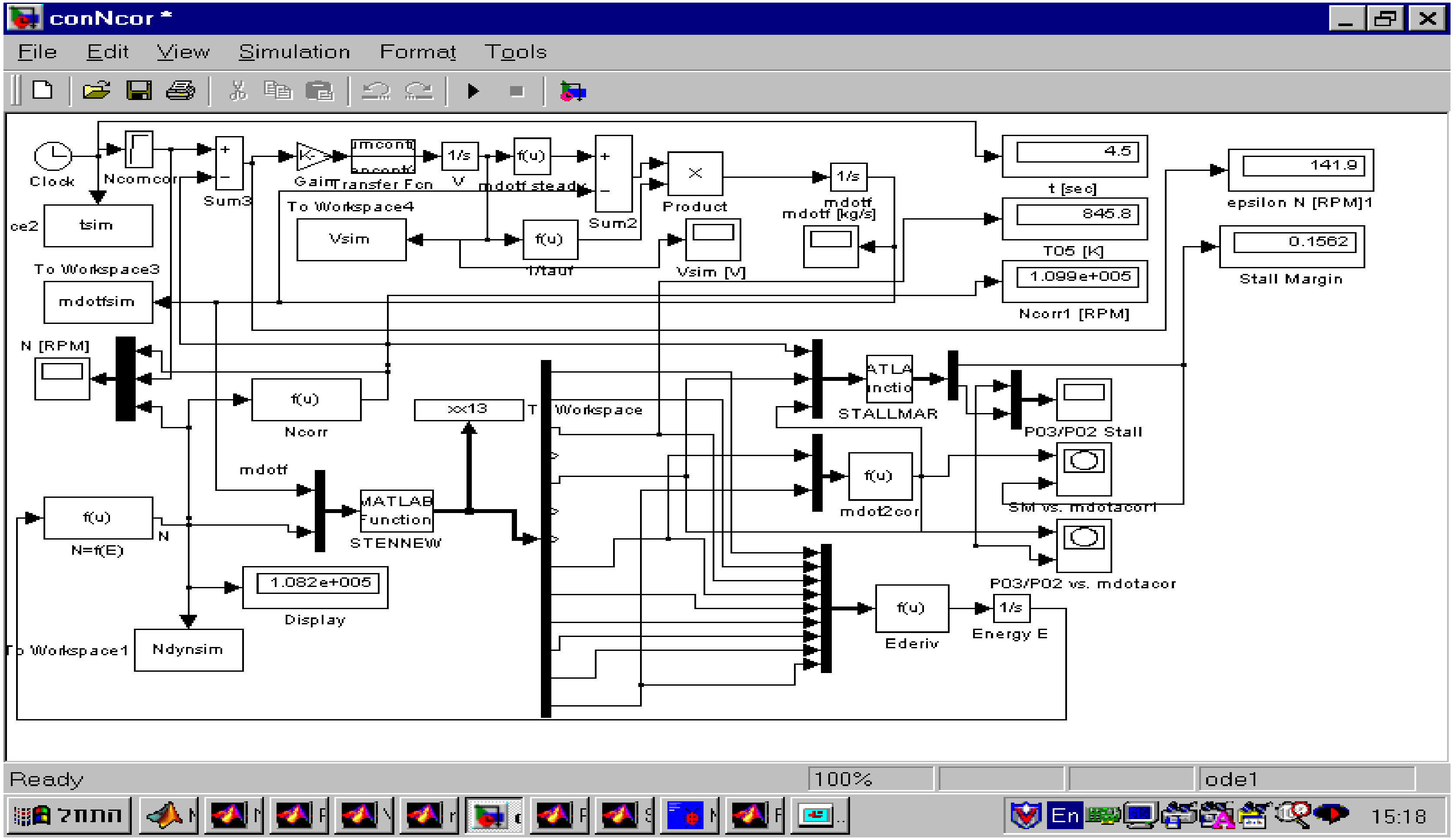
NONLINEAR CONTROLLER – T1 [s]:

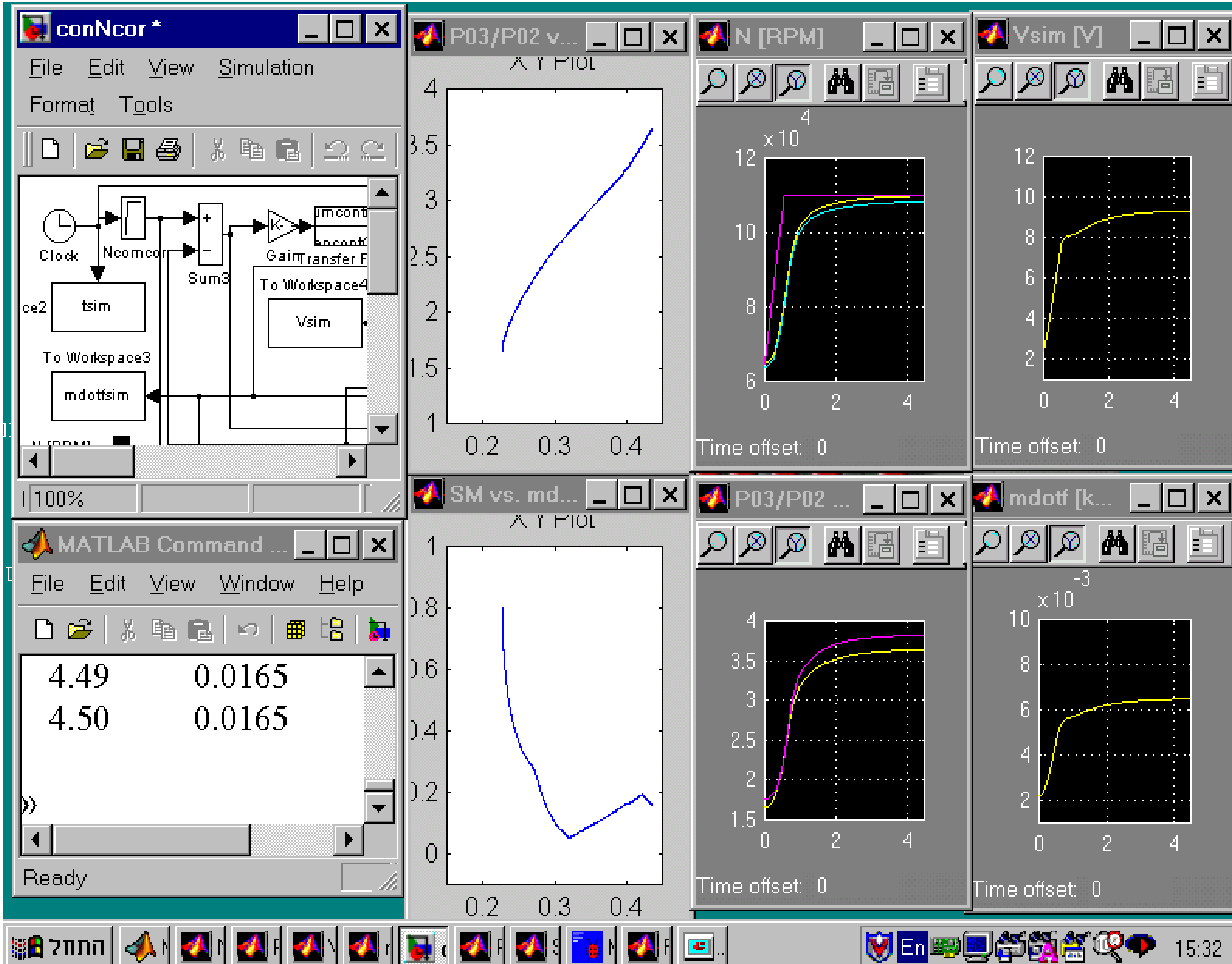
Hfeet / M	0	0.1	0.25	0.5	0.6	0.7
0	1.00	1.00	1.00	1.00	1.50	2.00
1000	0.70	0.60	0.80	0.80	1.40	1.70
2000	0.60	0.60	0.70	0.75	0.80	1.00
3000	0.40	0.40	0.40	0.40	0.40	0.50
4000	0.20	0.25	0.30	0.30	0.35	0.45
5000	0.20	0.20	0.25	0.30	0.35	0.40
7500	0.20	0.20	0.225	0.30	0.30	0.40
10000	0.20	0.17	0.20	0.30	0.30	0.40
15000	0.17	0.15	0.20	0.30	0.30	0.40
20000	0.15	0.15	0.20	0.30	0.30	0.40

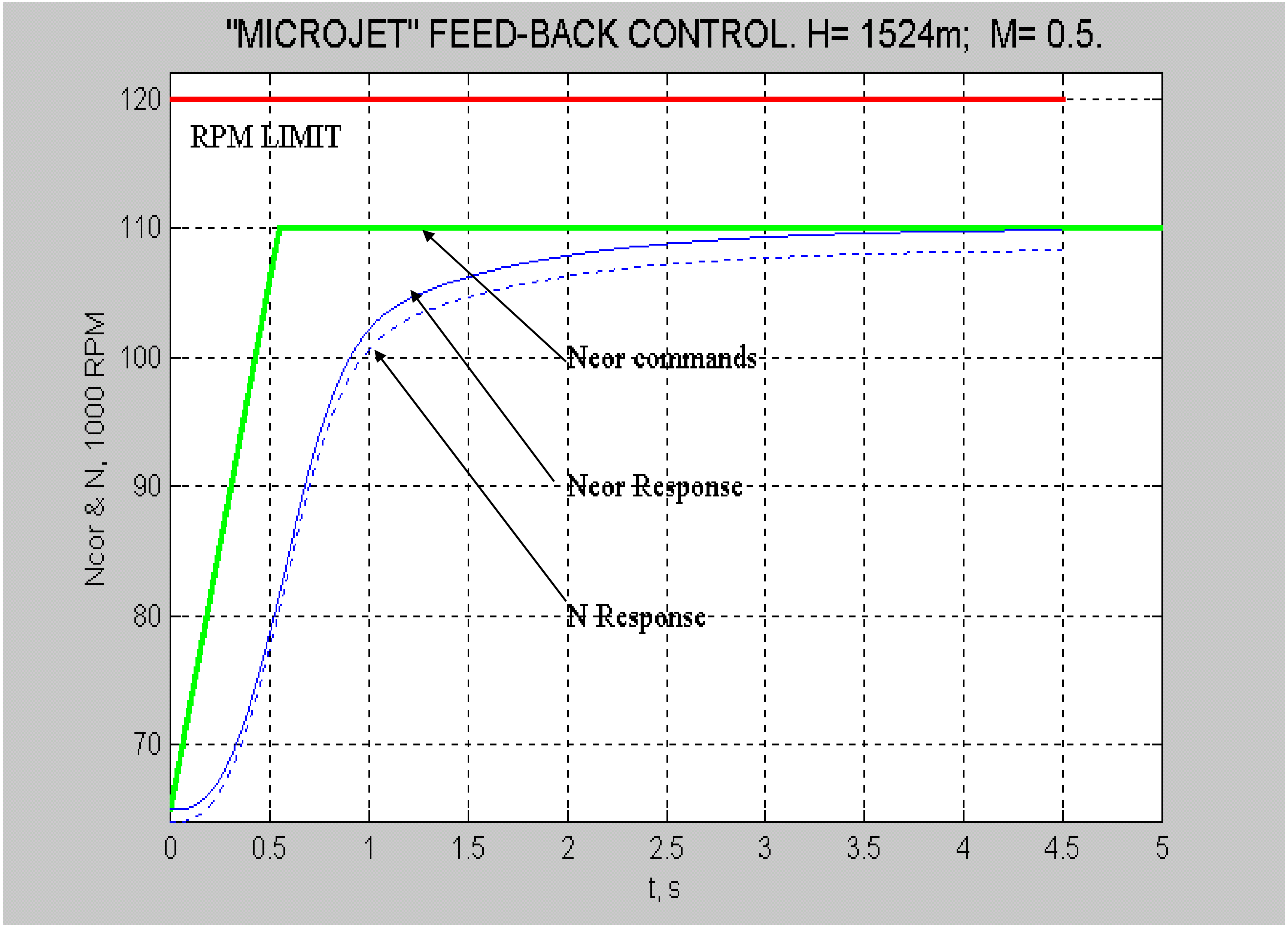
NONLINEAR CONTROLLER – K [V*s/RPM]:

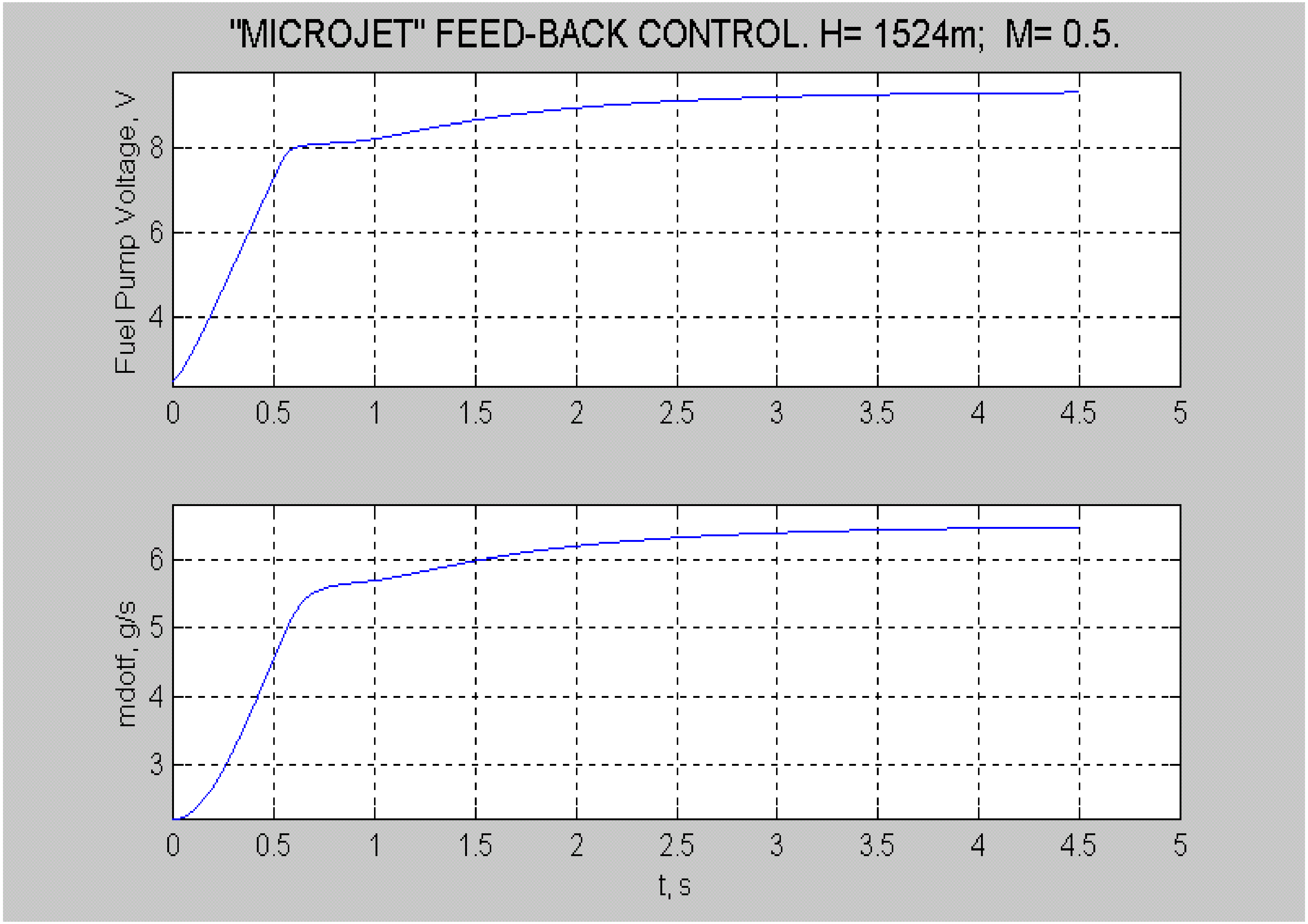
Hfeet / M	0	0.1	0.25	0.5	0.6	0.7
0	0.000100	0.00011	0.00014	0.00025	0.00050	0.00075
1000	0.000100	0.00012	0.00015	0.00025	0.00070	0.00100
2000	0.000075	0.00010	0.00016	0.00030	0.00073	0.00100
3000	0.000075	0.00010	0.00017	0.00035	0.00070	0.00150
4000	0.000075	0.00010	0.00015	0.00035	0.00070	0.00140
5000	0.000075	0.00010	0.00010	0.00030	0.00060	0.00050
7500	0.000075	0.00006	0.00010	0.00025	0.00040	0.00050
10000	0.000050	0.00006	0.00010	0.00020	0.00030	0.00040
15000	0.000030	0.00005	0.00007	0.00015	0.00017	0.00030
20000	0.000020	0.00003	0.00004	0.00010	0.00015	0.00020

SIMULINK PROGRAM FOR “MICROJET” CLOSE-LOOP SIMULATIONS

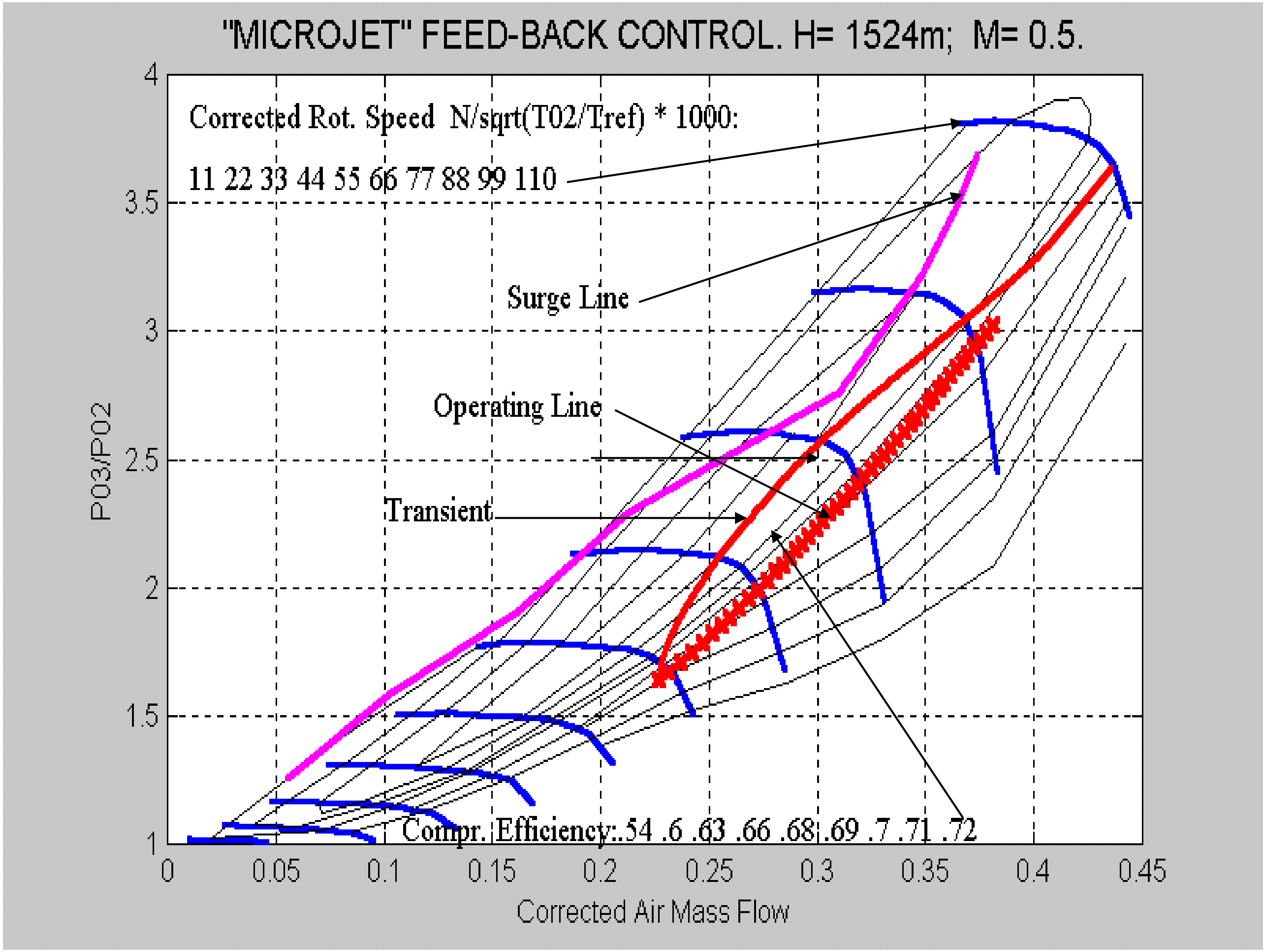


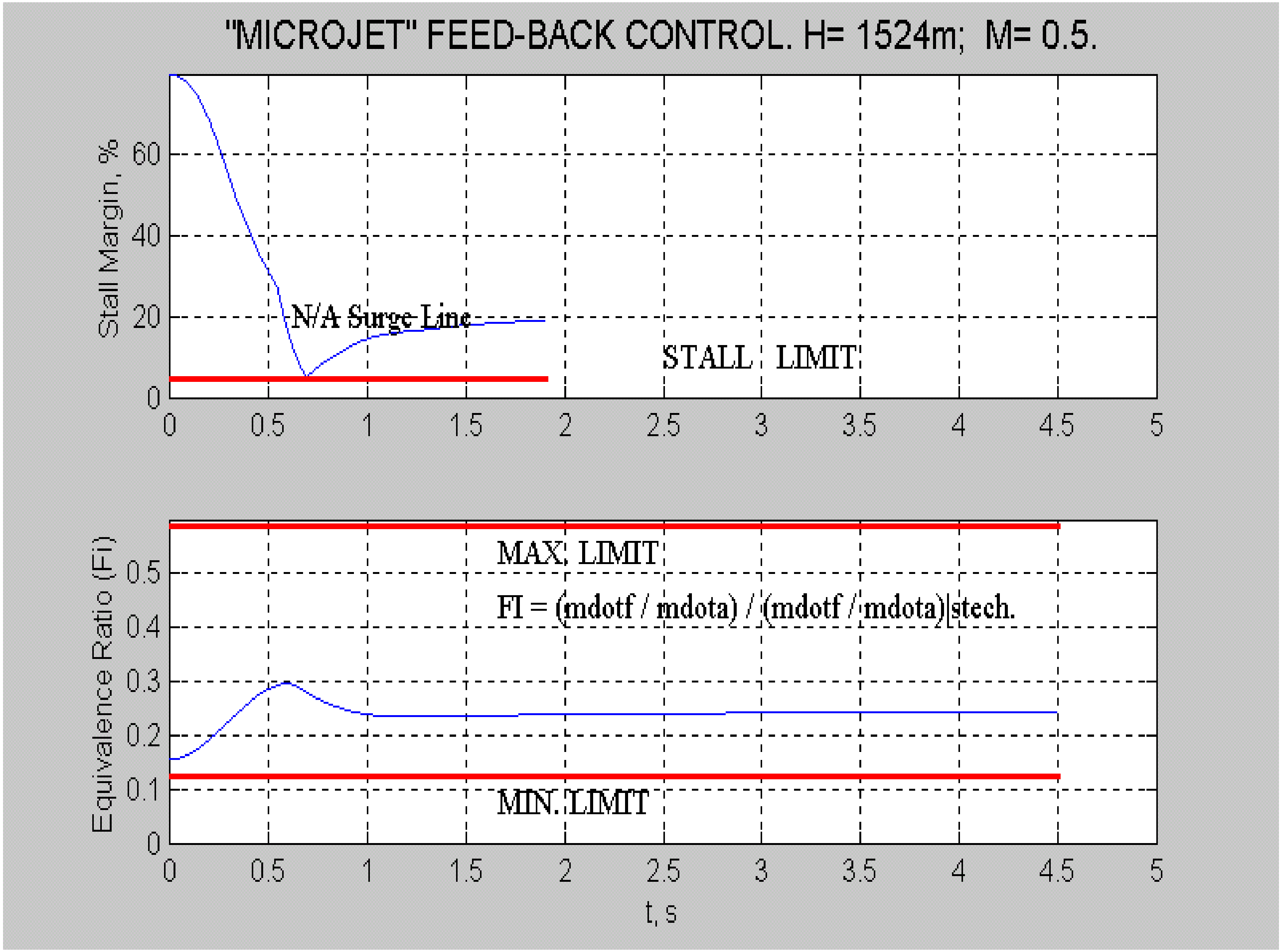

conNcor *
File Edit View Simulation
Format Tools
Clock
Ncomcor
Sum3
Gain
Transfer F
To Workspace4
tsim
Vsim
To Workspace3
mdotfsim
100%
P03/P02 v...
X Y Plot
N [RPM]
Time offset: 0
Vsim [V]
Time offset: 0
SM vs. md...
X Y Plot
P03/P02 ...
Time offset: 0
mdotf [k...
Time offset: 0
MATLAB Command ...
File Edit View Window Help
4.49 0.0165
4.50 0.0165
Ready
15:32

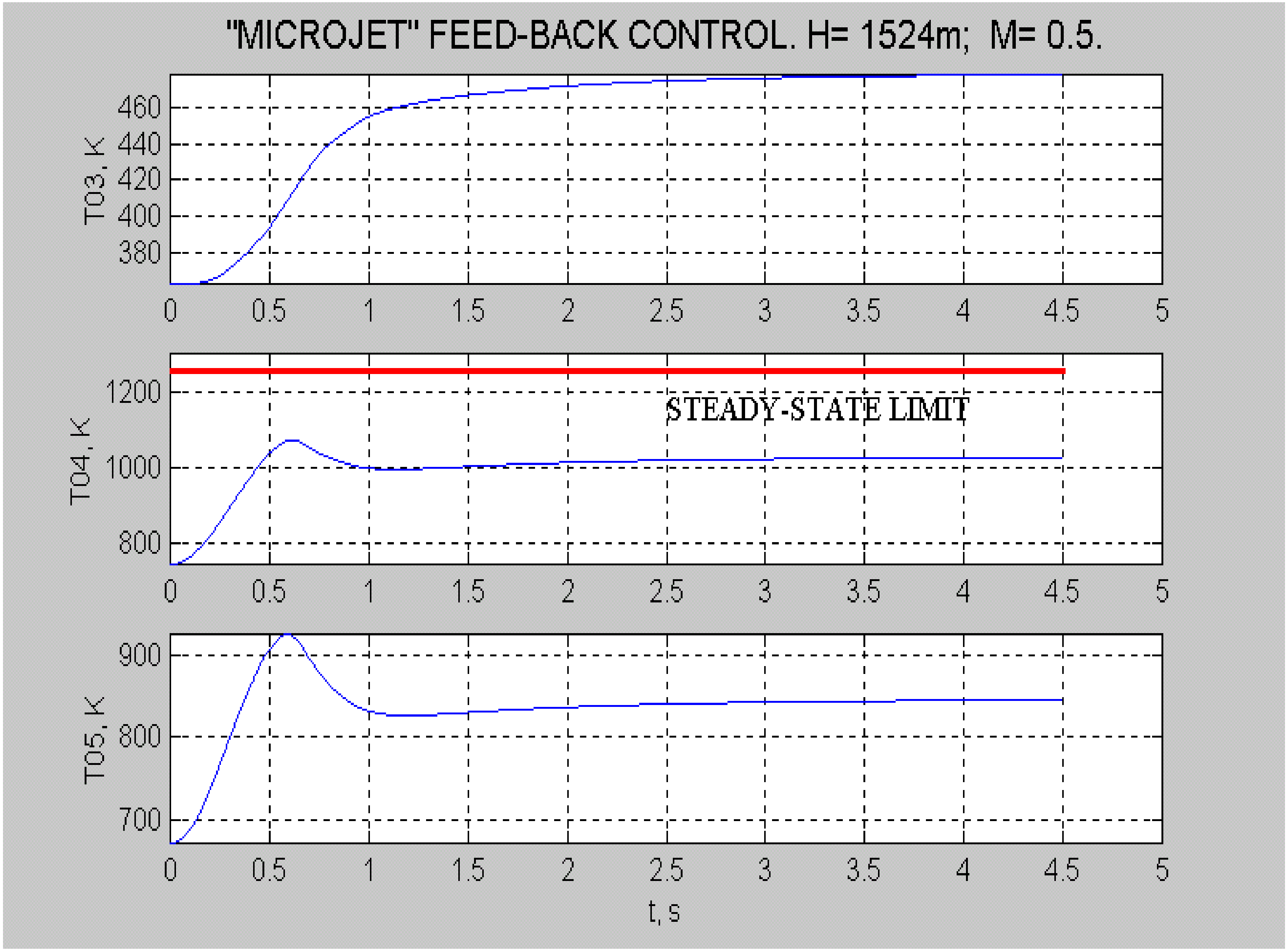

"MICROJET" FEED-BACK CONTROL. H= 1524m; M= 0.5.
RPM LIMIT
Ncor commands
Ncor Response
N Response
Ncor & N, 1000 RPM
t, s
120
110
100
90
80
70
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5

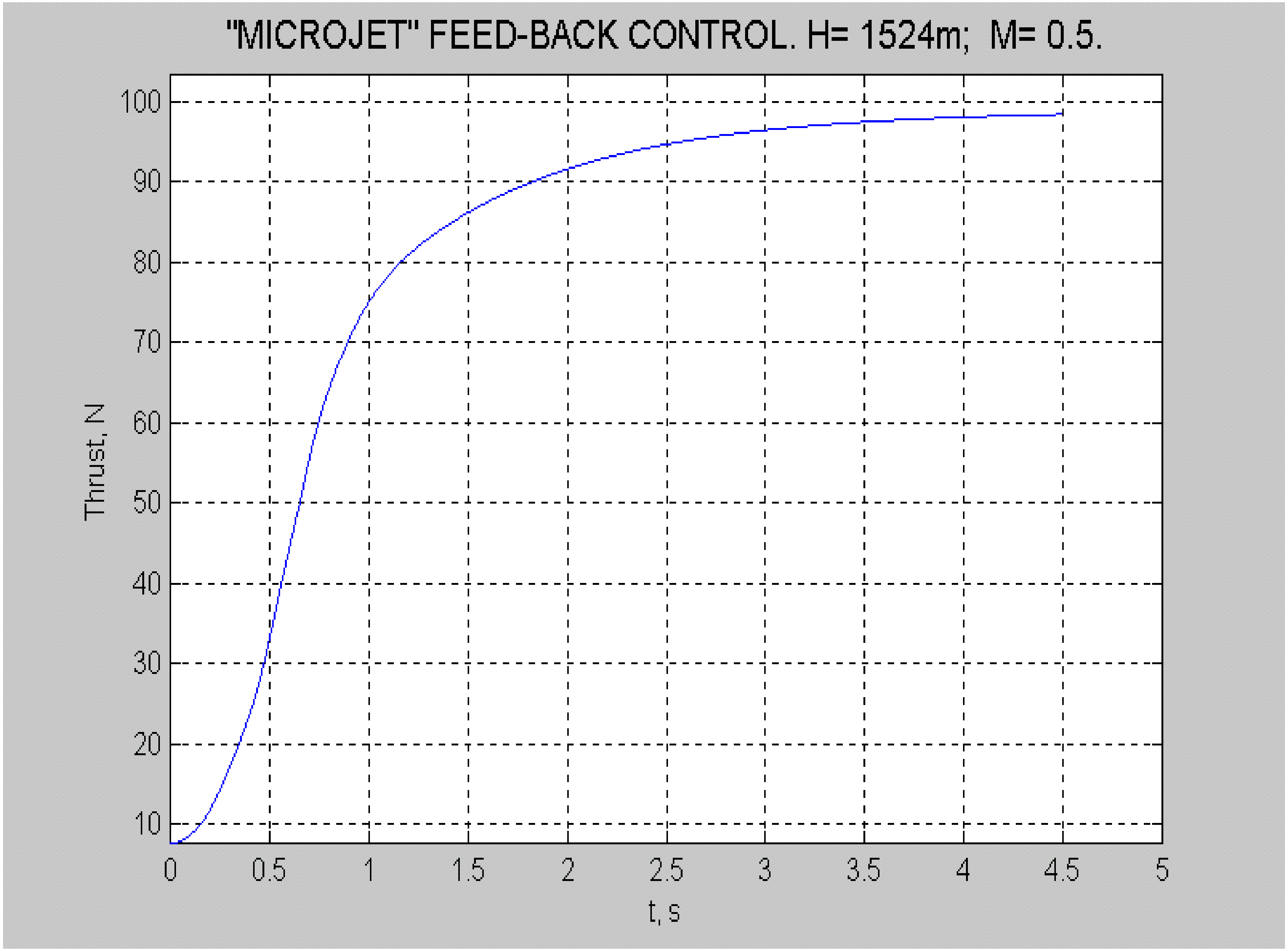


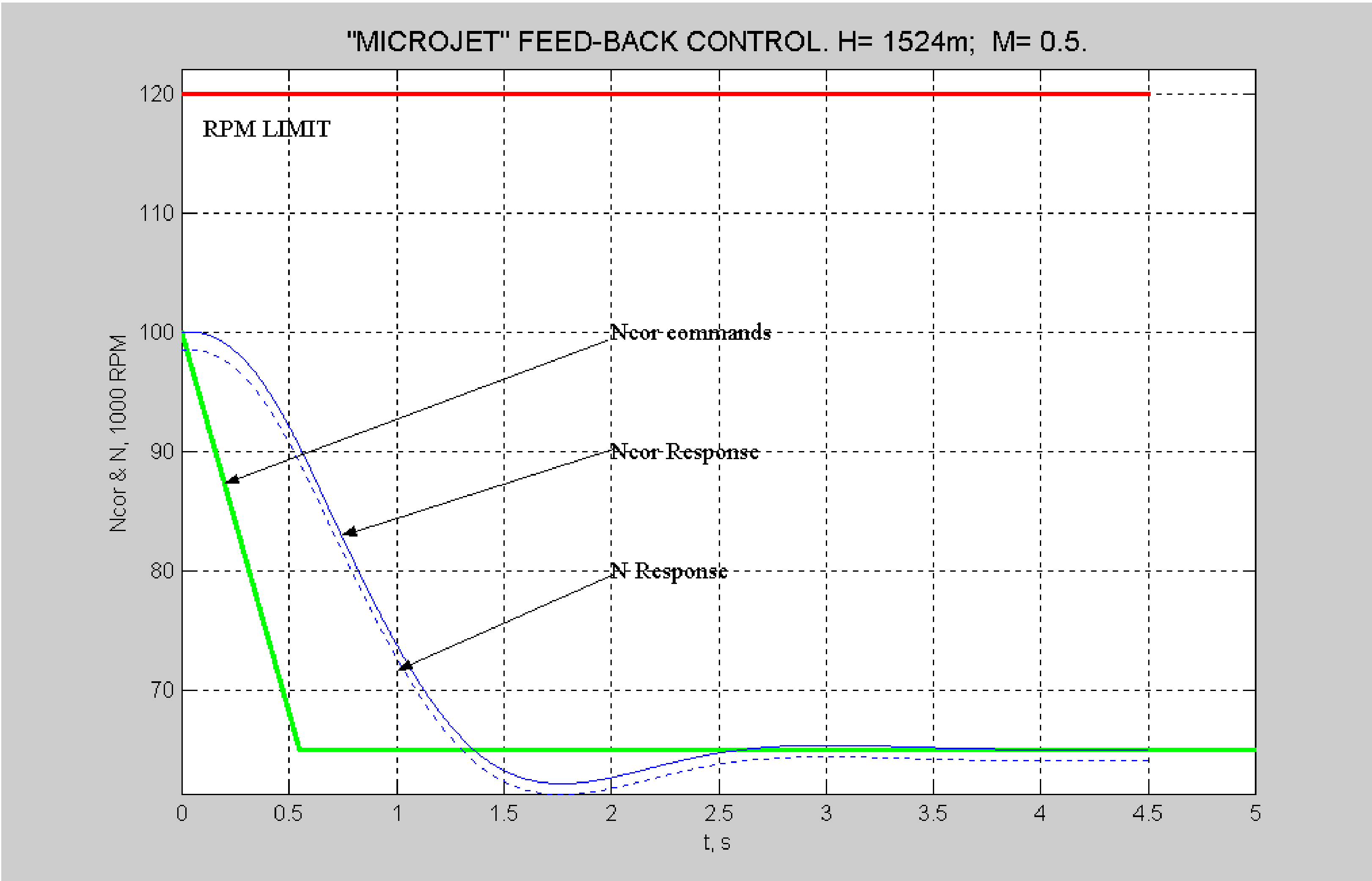
"MICROJET" FEED-BACK CONTROL. H= 1524m; M= 0.5.
Fuel Pump Voltage, V
4
6
8
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
mdotf, g/s
3
4
5
6
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
t, s


"MICROJET" FEED-BACK CONTROL. H= 1524m; M= 0.5.
Corrected Rot. Speed N/sqrt(T02/Tref) * 1000:
11 22 33 44 55 66 77 88 99 110
Surge Line
Operating Line
Transient
Compr. Efficiency:.54 .6 .63 .66 .68 .69 .7 .71 .72
P03/P02
Corrected Air Mass Flow
4
3.5
3
2.5
2
1.5
1
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45

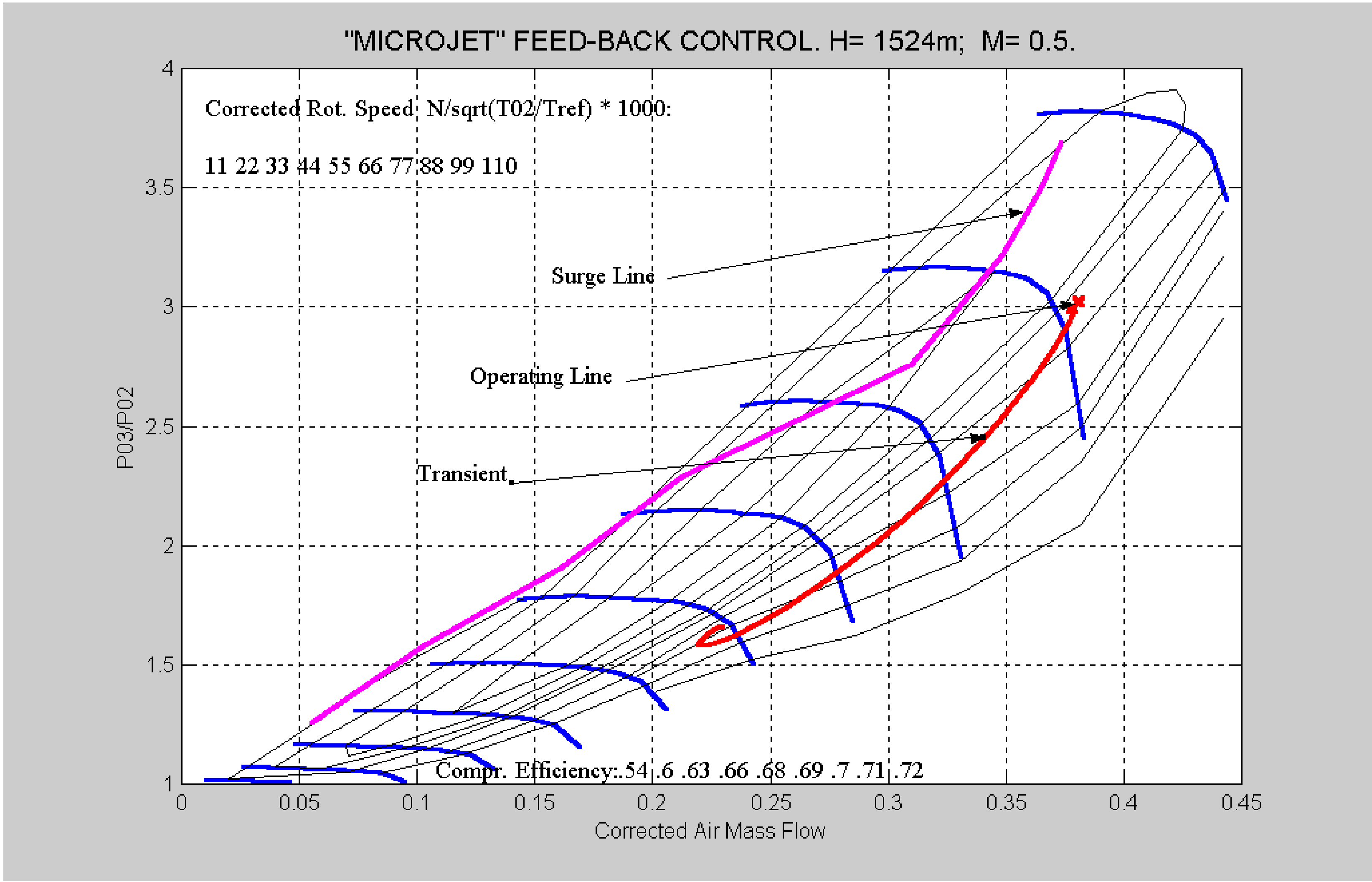

"MICROJET" FEED-BACK CONTROL. H= 1524m; M= 0.5.
Stall Margin, %
N/A Surge Line
STALL LIMIT
0
20
40
60
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
Equivalence Ratio (Fi)
MAX. LIMIT
FI = (mdotf / mdota) / (mdotf / mdota)|stech.
MIN. LIMIT
0
0.1
0.2
0.3
0.4
0.5
t, s

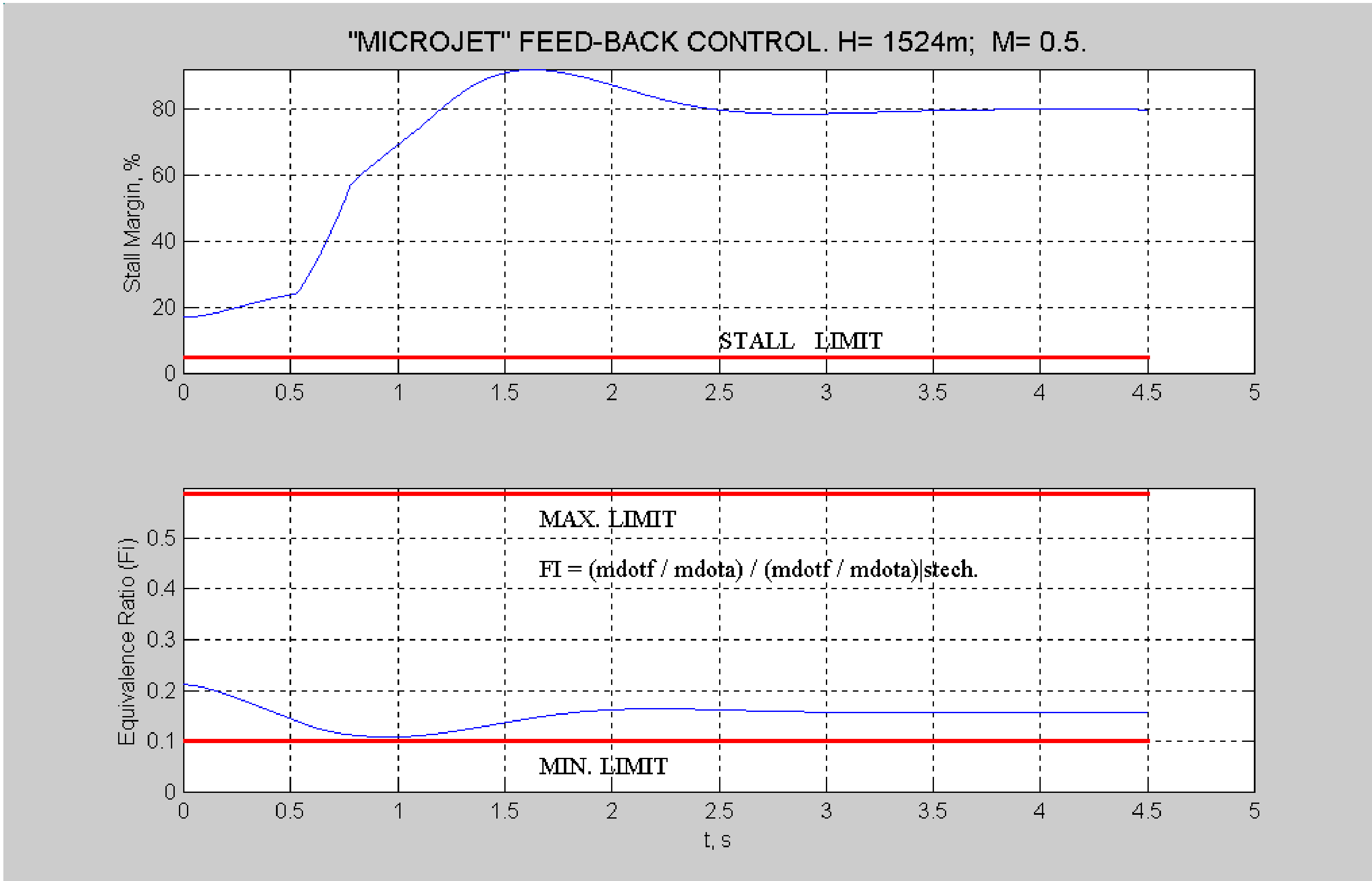

"MICROJET" FEED-BACK CONTROL. H= 1524m; M= 0.5.
T03, K
460
440
420
400
380
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
T04, K
1200
1000
800
STEADY-STATE LIMIT
T05, K
900
800
700
t, s


"MICROJET" FEED-BACK CONTROL. H= 1524m; M= 0.5.
100
90
80
70
60
50
40
30
20
10
Thrust, N
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
t, s



"MICROJET" FEED-BACK CONTROL. H= 1524m; M= 0.5.
RPM LIMIT
Ncor commands
Ncor Response
N Response
Ncor & N, 1000 RPM
t, s
120
110
100
90
80
70
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5


"MICROJET" FEED-BACK CONTROL. H= 1524m; M= 0.5.
Corrected Rot. Speed N/sqrt(T02/Tref) * 1000:
11 22 33 44 55 66 77 88 99 110
Surge Line
Operating Line
Transient
Compr. Efficiency:.54 .6 .63 .66 .68 .69 .7 .71 .72
P03/P02
Corrected Air Mass Flow
4
3.5
3
2.5
2
1.5
1
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45


"MICROJET" FEED-BACK CONTROL. H= 1524m; M= 0.5.
Stall Margin, %
STALL LIMIT
0
20
40
60
80
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
Equivalence Ratio (Fi)
MAX. LIMIT
FI = (mdotf / mdota) / (mdotf / mdota)|stech.
MIN. LIMIT
0
0.1
0.2
0.3
0.4
0.5
t, s

CONCLUSIONS:

1. DYNJET is a MATLAB/SIMULINK program for analysis of steady-state and dynamic performance of single-spool turbojet engine with convergent nozzle without afterburner
2. DYNJET is a flexible program. It allows adding new functions and equations for investigation purposes
3. Comparisons of simulation results with experimental data prove the model's validity.
4. 100 computer simulations at flight conditions: $0<H<20000$ ft and $0<M<0.7$ prove the nonlinear controller corresponds to the specification requirements