A slip factor calculation in centrifugal impellers based on linear cascade data

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Becker Turbo System Engineering (2005).
(Presented by Dr. E. Shalman)

Abstract

Accurate modeling of the flow slip against direction of rotation is essential for correct prediction of the centrifugal impeller performance. The process is characterized by a slip factor. Most correlations available for calculation of the slip factor use parameters characterizing basic impeller geometry (review of Wiesner (1967) and Backstrom (2006)).

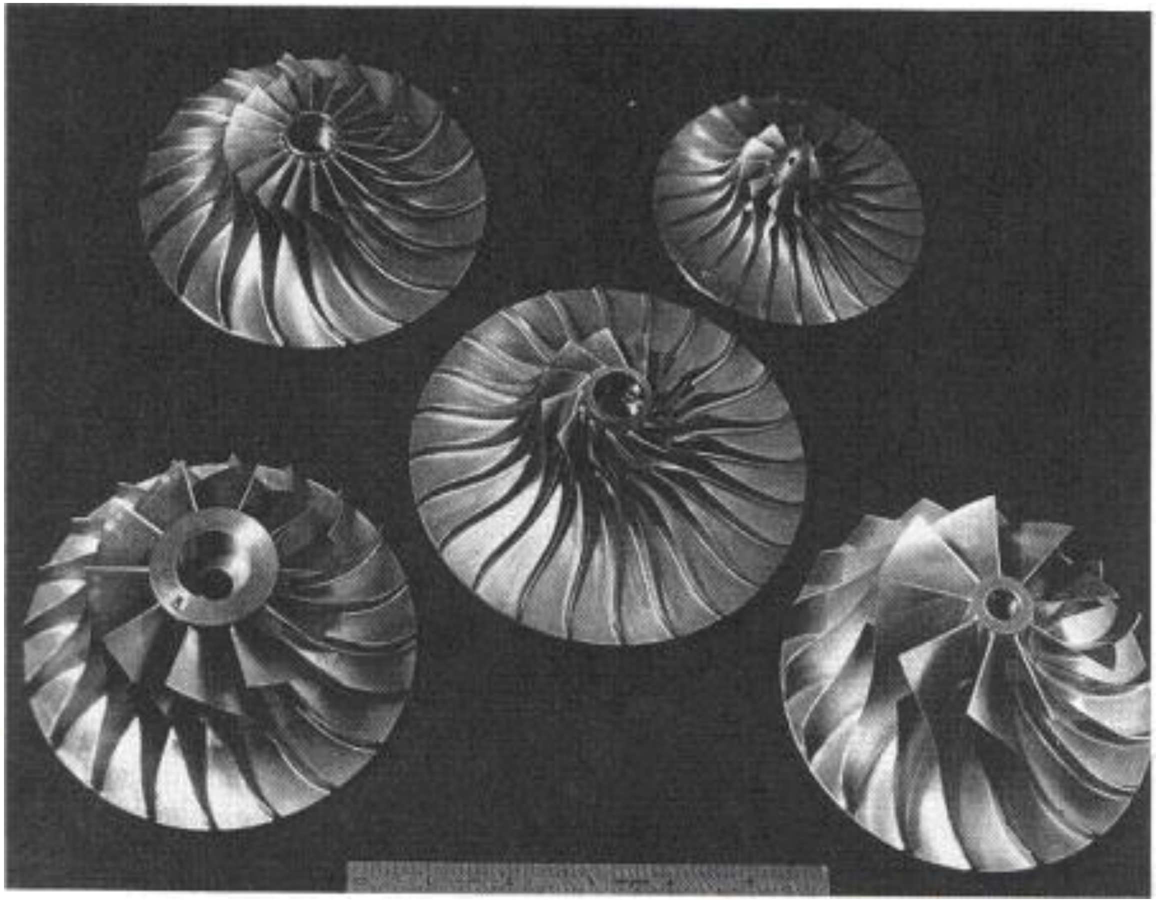
Approach presented below is based on reduction of radial cascade to equivalent linear cascade. The reduction allows to calculate characteristics of radial blade row using well established experimental data obtained for linear cascades, diffusers and axial blade rows.

Rotors with splitters or with high loading must be divided into few radial blade rows. In the case of multiple rotor blade rows the slip angle is calculated for each blade row. The slip angle of the rotor is a sum of the slip angles obtained for each row. Suggested reduction of radial blade rows allows also calculating of other parameters essential for impeller design.

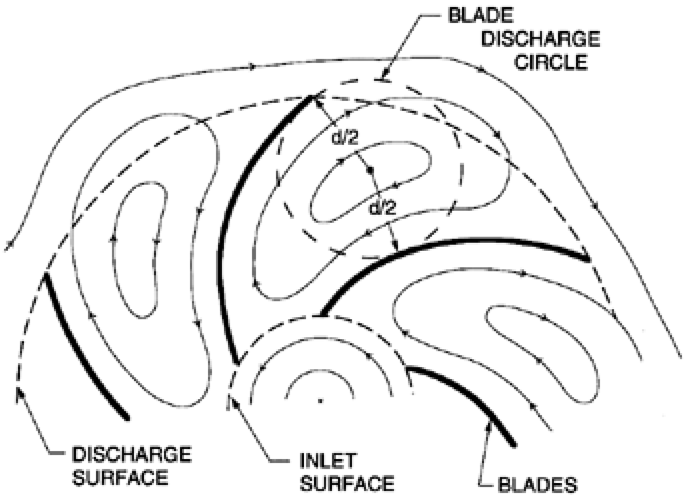
Suggested method allows also determine additional causes influencing slip factor. The slip factor depends not only on parameters characterizing basic impeller geometry, but on difference of inlet flow angle from stall flow angle. In the rotors with the same basic geometry parameters slip factor depends on the length of the blade. Slip factor increases with blade length.

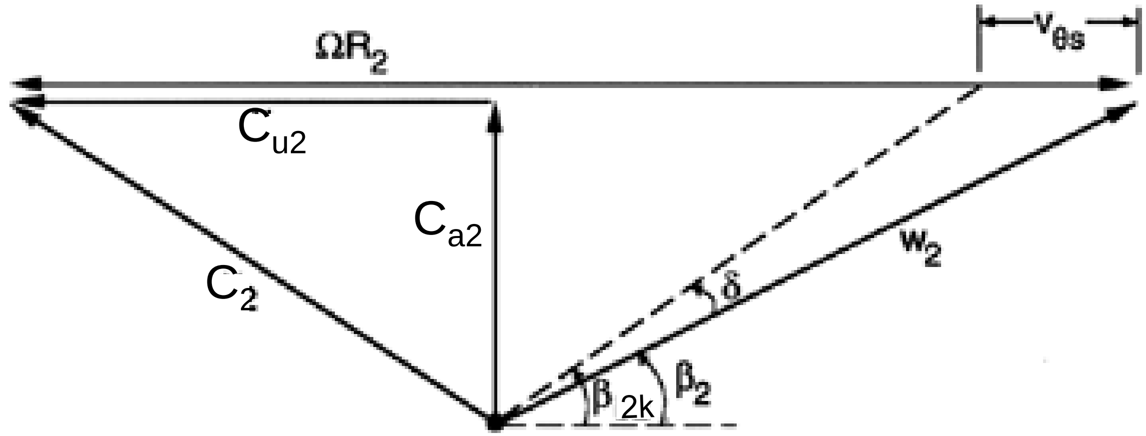
Axial compressor

Centrifugal compressor



Slip flow in Centrifugal compressor





$$\sigma = 1 - \frac{v_{\theta_s}}{\Omega R_2} \quad \text{(definition)}$$

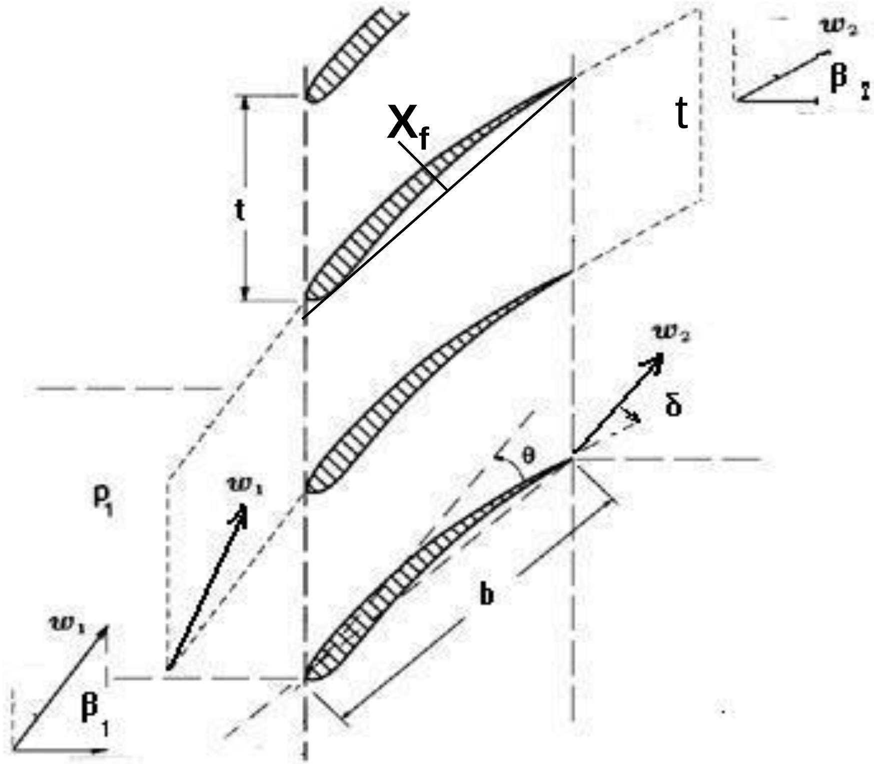
Busemann (1928) called this **displacement flow**; other authors refer to its rotating cells as **relative eddies**.

Stodola (1927)

$$v_{\theta_s} = \pi \ \Omega R_2 \sin \beta_{2k} \big/ Z$$

$$\sigma = 1 - \frac{\pi \ \sin \beta_{2k}}{Z}$$

Axial compressor

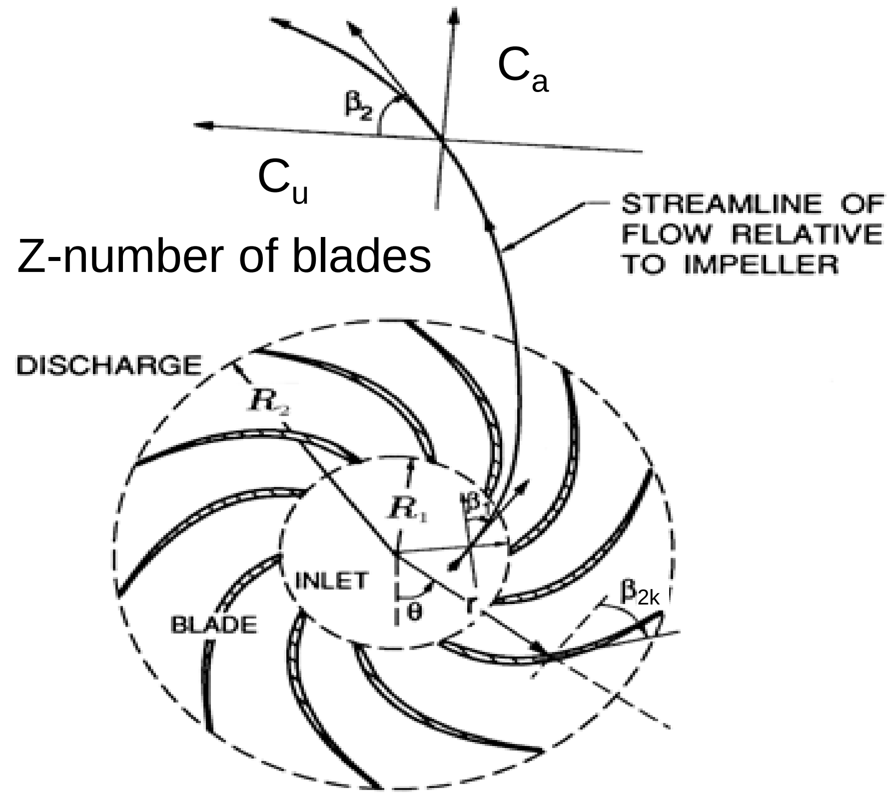


Howell (1945)

Deviation angle $\delta = \beta_2 - \beta_{2k}$.

$$\delta = \left(0.23\left(2\bar{x}_f\right)^2 + 0.002\left(\beta_{2k} + \delta\right)\right)\theta\sqrt{\bar{t}}$$

Centrifugal compressor

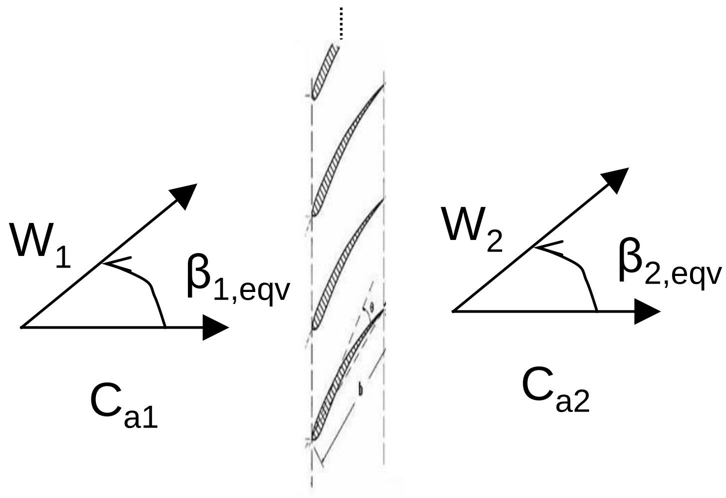


Slip factor σ or slip angle $\delta = \beta_2 - \beta_{2k}$.

$$\sigma = 1 - \frac{c_{a2}}{u_2}\left(\tan\left(\beta_{2k} + \delta\right) - \tan\left(\beta_{2k}\right)\right)$$

$$\sigma = 1 - \frac{\sqrt{\sin \beta_{2k}}}{Z^{0.7}}$$ (Wiesner)

$$\delta = \left(0.23\left(2\bar{x}_f\right)^2 + 0.002\left(\beta_{2k} + \delta\right)\right)\theta\sqrt{\bar{t}}$$



General Inviscid

$$\frac{\Delta P}{\rho_1 w_1^2/2} = 1 - \left(\frac{w_2}{w_1}\right)^2$$

In Linear cascade $C_{a1} = C_{a2}$

$$\frac{\Delta P_{eqv}}{\rho_1 w_1^2/2} = 1 - \left(\frac{w_2}{w_1}\right)^2 = 1 - \left(\frac{\cos(\beta_{1,eqv})}{\cos(\beta_{2,eqv})}\right)^2$$

General blade row

$$\frac{\Delta P}{\rho_1 w_1^2/2} = 1 - \left(\frac{w_2}{w_1}\right)^2 = 1 - \left(\frac{\cos(\beta_1)}{\cos(\beta_2)} K_D\right)^2$$

Equivalent linear cascade

$$\Delta P = \Delta P_{eqv}$$

$$\beta_2 = \beta_{2,eqv}$$

$$\cos(\beta_{1,eqv}) = K_D \cos(\beta_1)$$

$$\varepsilon_{eqv} = \beta_{1,eqv} - \beta_2$$

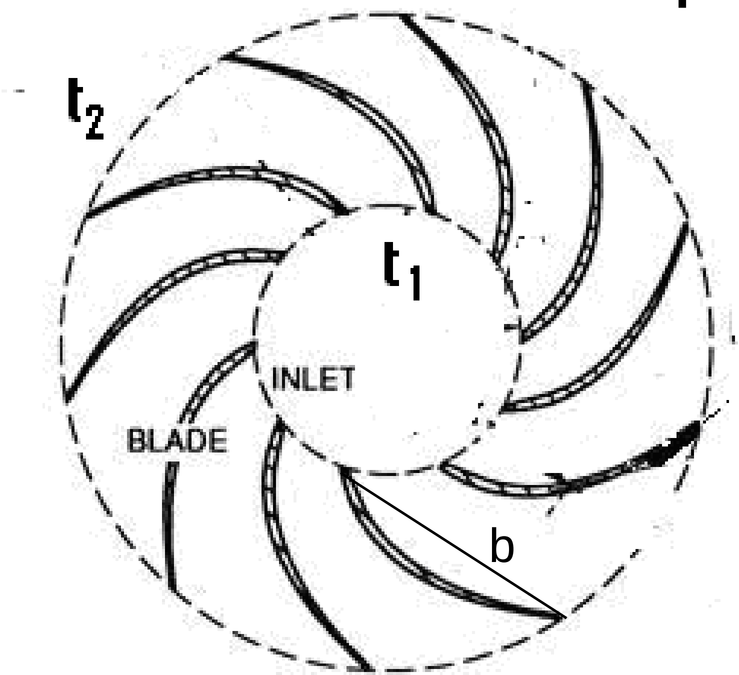
$$\varepsilon_{eqv} - i + \delta = \theta_{eqv}$$

$$\beta_2 = \beta_{2k} + \delta$$

$$\beta_1 = \beta_{1k} + i$$

$$\delta_{eqv} = \left(0.23\left(2\bar{x}_f\right)^2 + 0.002\left(\beta_{2k} + \delta\right)\right)\theta_{eqv}\sqrt{\bar{t}_{eqv}}$$

Calculation of equivalent pitch (solidity)



$$\bar{t}_{eqv} = \frac{2t_2^2}{\left(t_1 + t_2\right)_1 b} = \frac{t_2}{b}\frac{2t_2}{\left(t_1 + t_2\right)}$$

$$\beta_{2k,eqv} = \beta_{2k} + \left(0.23\left(2\bar{x}_f\right)^2 + 0.002\left(\beta_{2k} + \delta\right)\right)\theta_{eqv}\left(\sqrt{\bar{t}_{eqv}} - 1\right)$$

$$\sigma = 1 - \frac{c_{a2}}{u_2}\left(\tan\left(\beta_{2k,eqv} + \delta_{eqv}\right) - \tan\left(\beta_{2k,eqv}\right)\right)$$

$$\cos\left(\beta_{1,eqv}\right) = K_D \cos\left(\beta_1\right)$$

Calculation of the coefficient K_D

1. Influence of the channel height h.

Stratford (1959) obtained the height (h) of the diffuser with given length (b) that has maximal static pressure rise coefficient. The velocity ratio w_2/w_1 depends on the ratio b/h. The experimental data for linear cascade were obtained for h=2b. The experimental data of Stafford may be approximated by equation

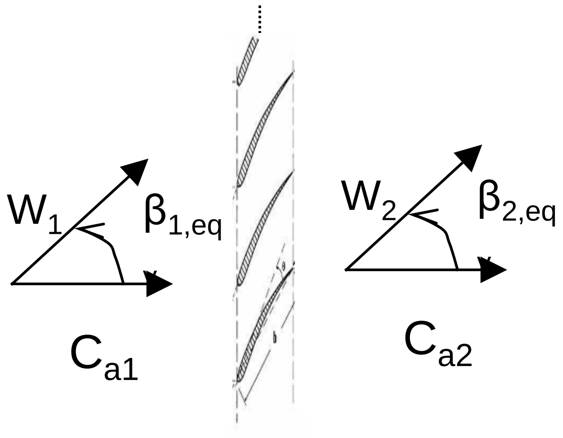
Experimental data for diffusers

$$\frac{\left(w_2 / w_1\right)}{\left(w_2 / w_1\right)_{h=2b}} = \left(\frac{2b}{h}\right)^{0.1}$$

$$\cos\left(\beta_{1,eqv}\right) = K_D \cos\left(\beta_1\right)$$

Calculation of the coefficient K_D

2. Influence of the ratio C_{a1}/C_{a2}.



Cascade with constant height and solidity:

$$\frac{w_2}{w_1} = \frac{c_{a2}}{c_{a1}} \frac{\cos\left(\beta_1\right)}{\cos\left(\beta_2\right)}$$

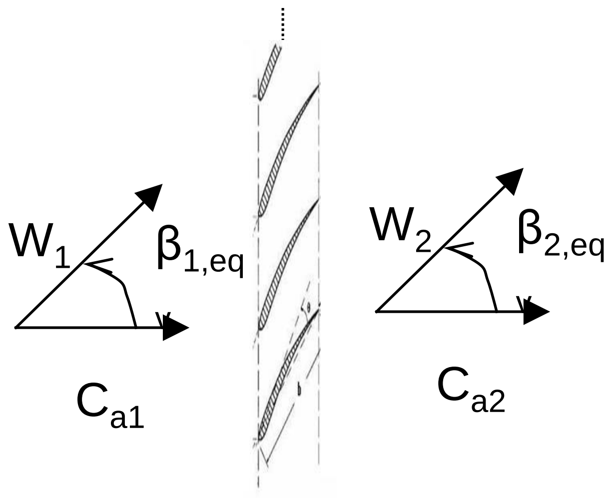
If the solidity is changed, then velocity ratio in the equivalent cascade must be multiplied by the pitch ratio

$$\frac{w_2}{w_1} = K_T \frac{\cos\left(\beta_1\right)}{\cos\left(\beta_2\right)}$$

$$K_T = \frac{c_{a2}}{c_{a1}} \frac{t_1}{t_2} \left(\frac{2b}{h}\right)^{0.1}$$

$$\cos\left(\beta_{1,eqv}\right) = K_D \cos\left(\beta_1\right)$$

Calculation of the coefficient K_D



For axial compressors

$$\frac{w_2}{w_1} = \frac{\cos\left(\beta_{1,eqv}\right)}{\cos\left(\beta_{2,eqv}\right)} = K_T \frac{\cos\left(\beta_1\right)}{\cos\left(\beta_2\right)}$$

where $K_T = \frac{c_{a2}}{c_{a1}} \frac{t_1}{t_2} \left(\frac{2b}{h}\right)^{0.1}$ Hence, $K_T = K_D$

In General case

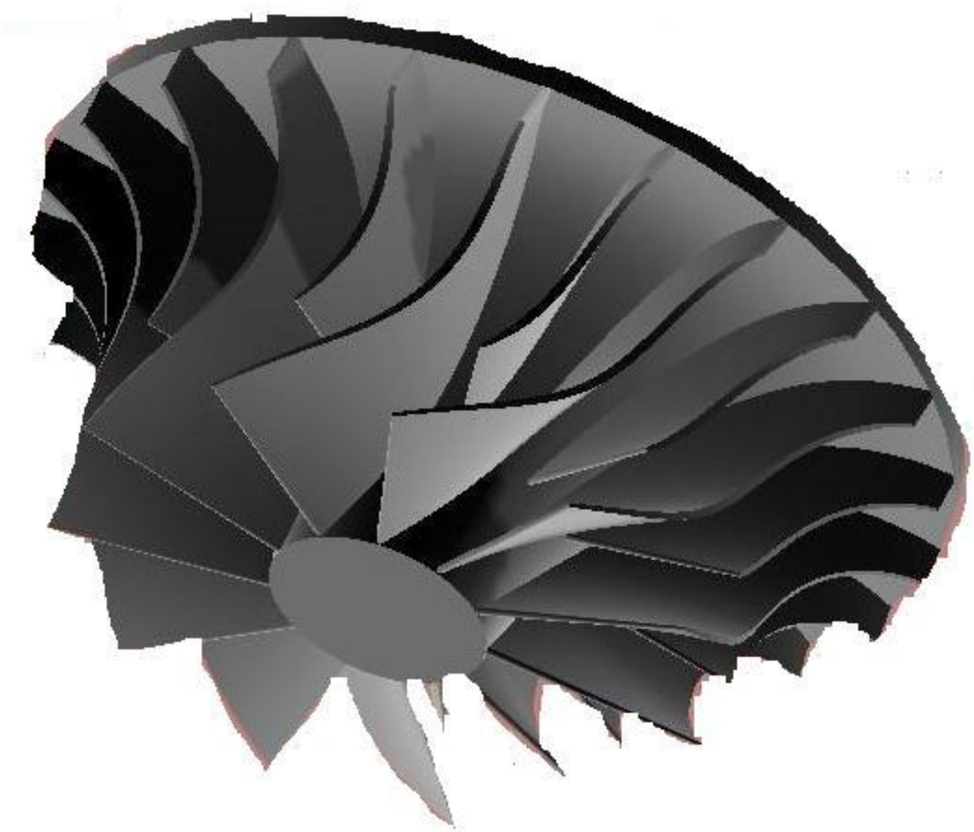
$$K_D = K_T^{\frac{1}{2K_T}}$$

where

$$K_T = \frac{c_{a2}}{c_{a1}} \frac{t_1}{t_2} \left(\frac{2b}{h}\right)^{0.1}$$

(effect of boundary layer separation and blockage)

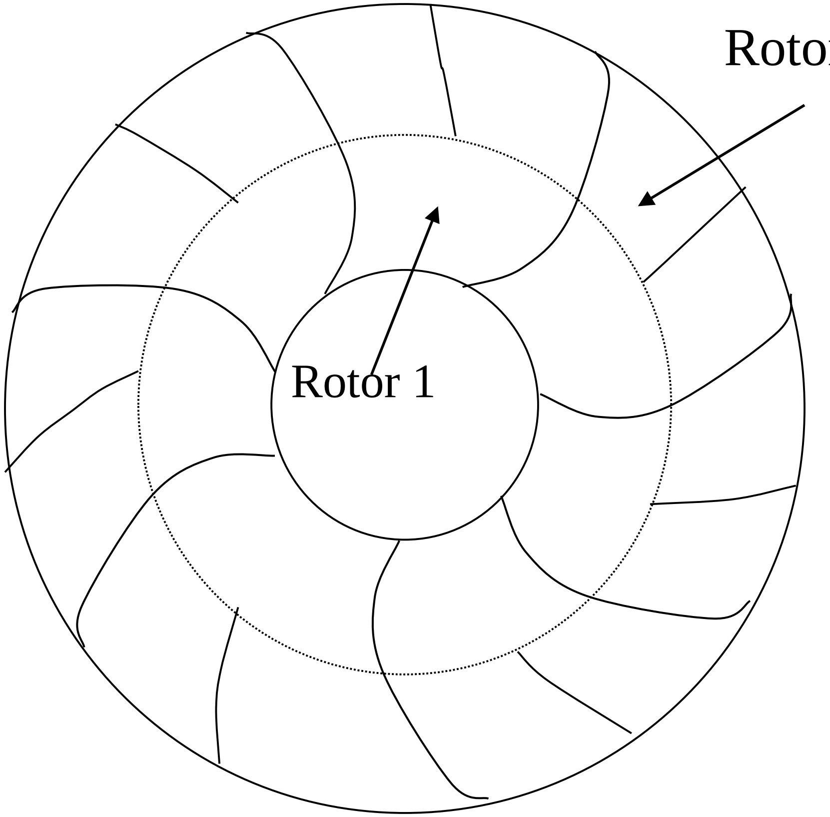
Impellers with high blade loading and splitters



The rotor with splitters is divided into sequence of rotors. Each rotor has the same number of blades at inlet and exit.

For high loaded rotors the loading of each equivalent linear cascade must be less then maximal allowed.

Impellers with high blade loading and splitters



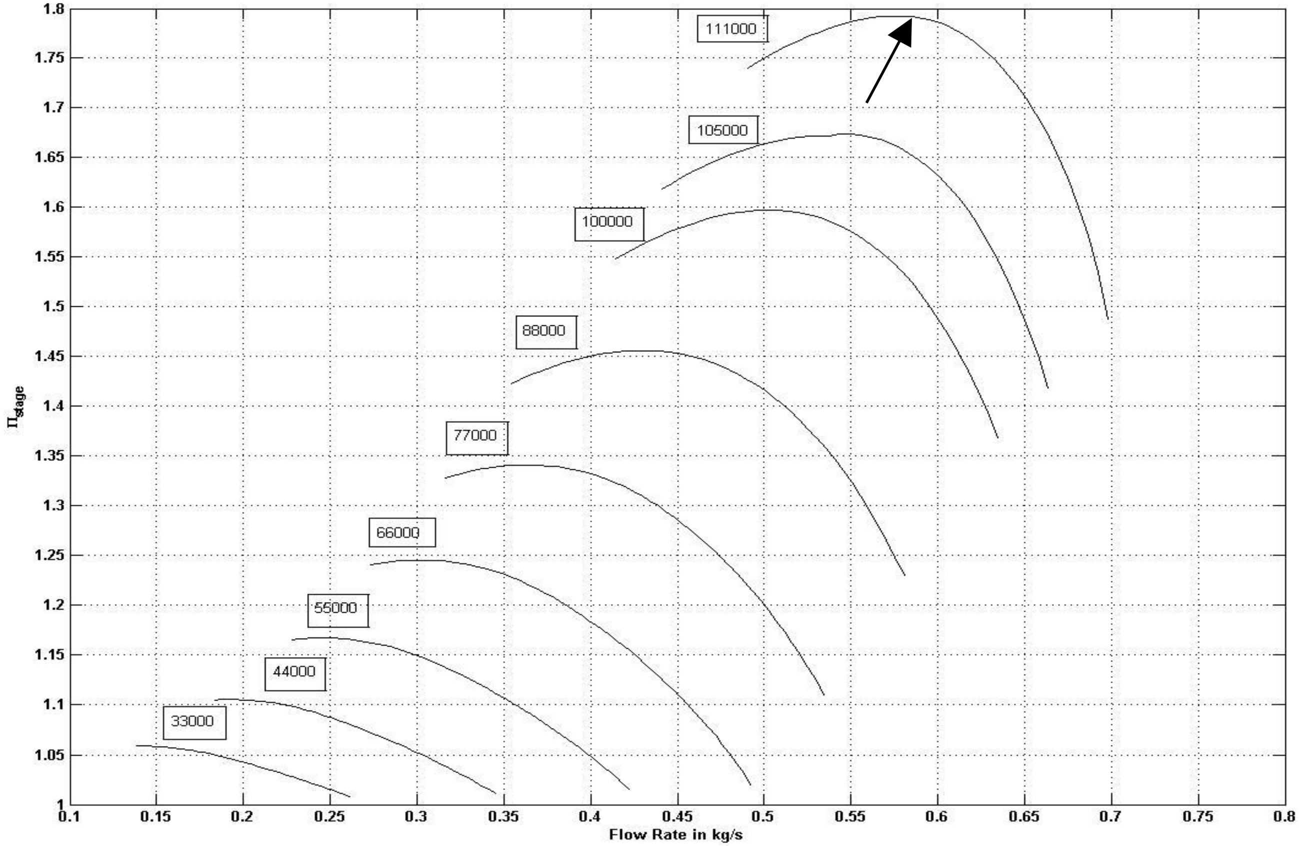
Calculate slip angle for each rotor.

The slip angle of the rotor is a sum of the slip angles obtained for each row

High blade loading

$$\cos\left(\beta_{1,eqv}\right) = K_D \cos\left(\beta_1\right) \qquad K_D = K_T^{\frac{1}{2K_T}} \qquad K_T = \frac{c_{a2}}{c_{a1}} \frac{t_1}{t_2} \left(\frac{2b}{h}\right)^{0.1}$$

Stall conditions



Maximal pressure gradient

$$\cos\left(\beta_{1,eqv}\right) = K_D \cos\left(\beta_1\right) \qquad K_D = K_T^{\frac{1}{2K_T}} \qquad K_T = \frac{c_{a2}}{c_{a1}} \frac{t_1}{t_2} \left(\frac{2b}{h}\right)^{0.1}$$

Analysis of experimental data of Howell (1945), Emery (1957), Bunimovich (1967)allows obtaining following equation for coefficient of static pressure rise at stall conditions (maximal pressure rise)

$$\frac{\Delta P_{st}}{\rho_1 w_1^2 / 2} = 1 - \left(\frac{w_{2,st}}{w_{1,st}}\right)^2 = 1 - \left(\frac{\cos\left(\beta_{1eqv,st}\right)}{\cos\left(\beta_{2eqv,st}\right)}\right)^2 = \frac{1.124}{1 + m_{st}\bar{t}}$$

$$m_{st} = \int_0^b \left(w_{SuctionSide} - w_1\right) db \Big/ \int_0^b \left(w_1 - w_{PressureSide}\right) db$$

Parameter m_{st} characterizes type of the cascade. E.g. For cascades studied by Howell m_{st} =1.

$\bar{t} = t / b$ is relative pitch of the linear cascade

Compressor Stages used to test the model

1. Stage of turbojet Olympus
2. Stage tested by Stechkin
3. Stage designed by Beker Engineering and tested at Concept.
4. Stage designed by Beker Engineering , not tested
5. Stage C1 (tandem blades) (data from USSR)
6. Monig et al. Trans ASME. Journal of turbomachinery 1993,v115, p.565-571.
7. Stage C9 (data from USSR)
8. Musgrave D., Plehn N.J. Mixed flow stage design and test results with a pressure ratio of 3:1. ASME Gas turbine conference presentation 87-GT-20.

Results

Table 1. Rotor parameters.

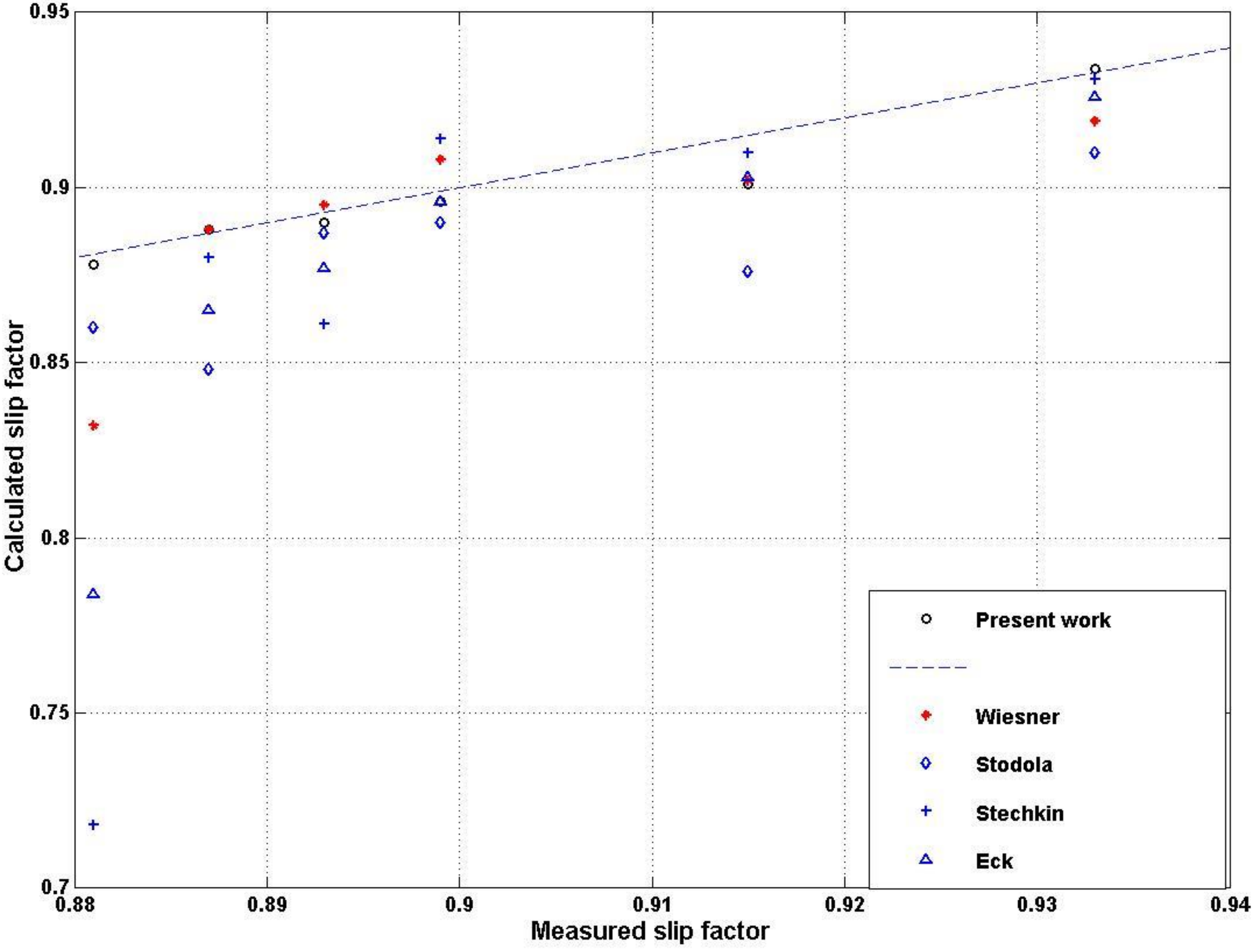
Rotor No.	**Z** number of blades	Rotor pressure ratio	**G** Kg/s	β_{2k} (outlet blade angle)	$\frac{r_2}{r_1}$	$t_{1,mean}$ (inlet relative pitch)	$t_{2,mean}$ (outlet relative pitch)	β_1 (inlet flow angle)	c_{a2} m/s	u_2 m/s (tip velocity)
1	6/12	4.35	0.41	16.99	1.83	1.0	0.695	57.8	180	472.3
2	12	3.95	3.0	33.58	2.30	0.50	-	32.7	170	483
3	11/22	5.23	0.48	12.0	1.89	1.27	0.397	57.48	189.3	500
4	7/14	3.87	1.0	48.28	2.1	1.50	0.641	56.33	182.9	550
5	27/27	4.54	2.7	9.0	2.12	0.25	0.346	46.65	185.3	455
6	15/30	3.86	2.0	8.48	2.0	0.74	0.316	50.47	160	425
7	18/36	6.79	2.75	6.25	2.18	0.56	0.175	46.25	161	520
8	12/24	6.36	2.89	19.9	1.78	1.0	0.612	60.05	165	510.6

Results

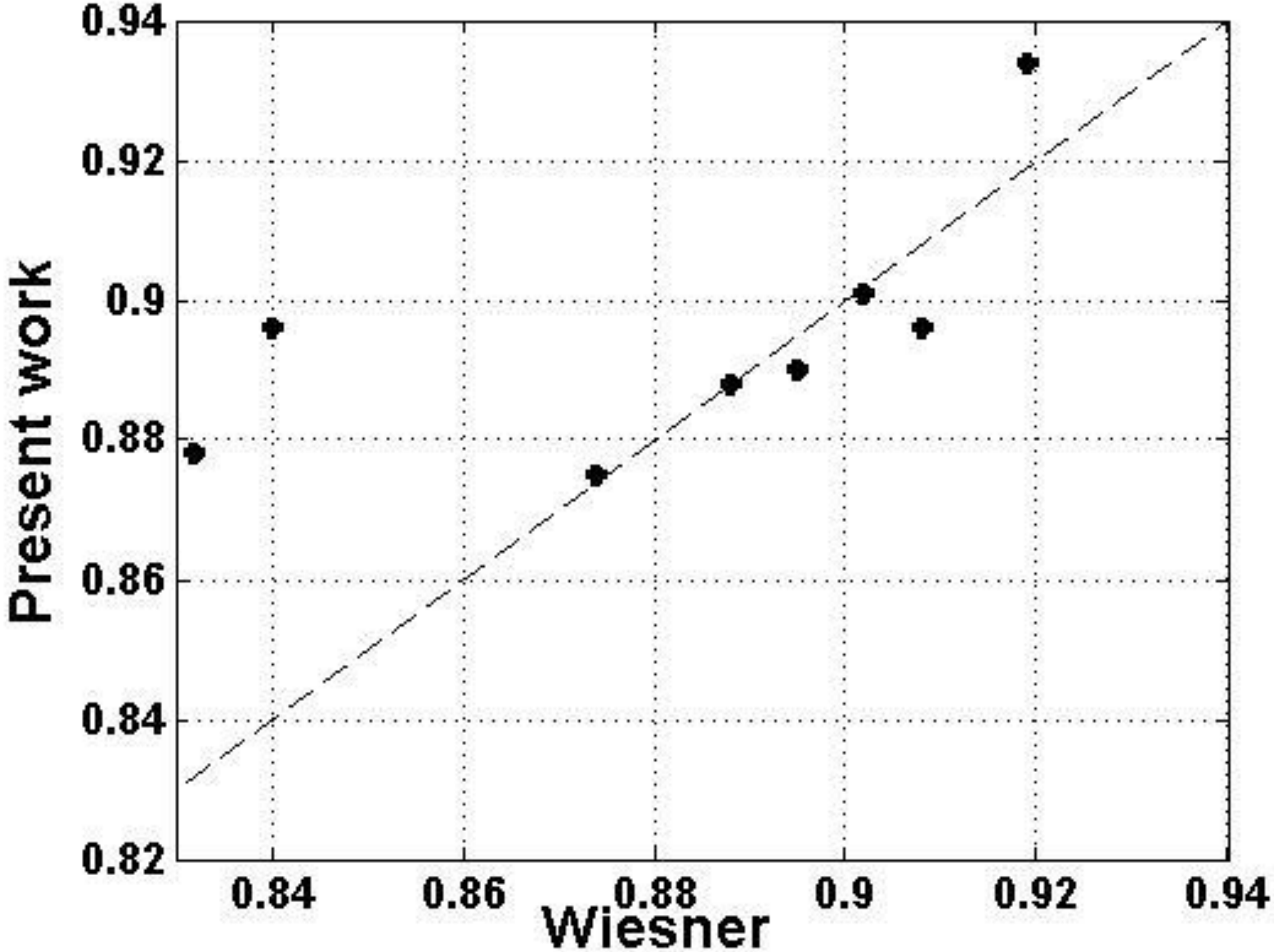
Table 2. Slip factor and exit angle.

Rotor No.	β_2 measured	β_2 Present work	σ measured	σ Present work	σ Wiesner	σ Stodola	σ Stechkin	σ Eck
1	31.8	32.0	0.881	0.878	0.832	0.860	0.718	0.784
2	-	43.8	-	0.896	0.840	0.72	0.823	0.840
3	27.0	27.0	0.887	0.888	0.888	0.848	0.880	0.865
4	-	56.3	-	0.875	0.874	0.762	0.834	0.870
5	20.17	20.6	0.915	0.901	0.902	0.876	0.910	0.903
6	23.32	23.0	0.899	0.896	0.908	0.890	0.914	0.896
7	17.9	17.8	0.933	0.934	0.919	0.910	0.931	0.926
8	35.9	35.0	0.893	0.890	0.895	0.887	0.861	0.877

Measured and calculated slip factor for 6 impellers



Calculated slip factor for 8 impellers



Conclusions

- Suggested method allowing reduction of arbitrary rotor blade passage to equivalent linear cascade
- Method allows establishing equivalence between axial and centrifugal impellers
- Method allows calculating slip factor based on well established linear cascade correlations for deviation angle