When the critical rotation speed of a flexible rotor lies within the working rotation speed margins, crossing it becomes a major problem for the flexible rotor’s operation.

The system investigated below is a rotor with two supports having a critical rotation speed within the working range. The system is shown in slide 3. First a simulation was performed for the system being supported by ball bearings only, without elastic rings around them.
The stiffness of the supports is the radial stiffness of the bearings.
A damping $\zeta=0.01$ was assumed.

Following this simulation, the rotor's center of gravity frequency response graph was constructed. The frequency response diagram of the c.g. is shown in slide 4.

The maximal bearing reaction, when in resonance, is 5kgf.
This can be seen in slide 5.
MAXIMAL BEARING REACTION

Bearing Reaction, kg

Rotation speed, krpm
The simulation was performed using MSC\Nastran for Windows. In this software package, it is not possible to perform a rotor dynamics analysis. Therefore, the bearing reactions were calculated by formula. The formulae used for the calculation of the bearing reactions are:

**TOTAL FORCE ON SUPPORTS FROM REVOLVING ROTOR**

\[ Q_i = (e_o + T2_i) \cdot M \cdot \omega_i^2 \]

Where
- \( e_o \) - Residual specific unbalance (\( e_o = \text{Unbalance} / \text{Rotor Weight} \))
- \( T2_i \) – Rotor center gravity translation under unbalance
- \( M \) - Rotor mass
- \( \omega_i \) - Rotor rotating speed
SUPPORT REACTION

\[ R1_i = \frac{L}{l} Q_i; R2_i = Q_i - R1_i \]

\[ R2_i < R1_i \]

Where

\( L \) and \( l \) are Rotor Geometrical Parameters:

\( L \)-maximal distance between Rotor center gravity and support

\( l \) – distance between bearings

Operation with a 5 kgf load is not possible: when in resonance the center of gravity oscillates at 20 mm/s. This value is too high for the system in question.
Our goal is to go through the resonant frequency. For this purpose, elastic rings, known as Allisson’s dampers, are fitted around the bearing.

As a rule, the bearing with the highest reaction load is the bearing fitted with an Allisson’s damper.

Those elastic rings have alternating internal and external protuberances.

An Allisson’s damper is shown in the figure in slide 9.
With the geometry of the ring known, its stiffness can be calculated.

The formula for the **Allisson’s damper** radial stiffness is:

\[
\frac{1}{k} = \frac{(D_{av} - 0.3 \cdot b_1 \cdot z)^3}{0.129 \cdot b \cdot E \cdot z^4 \cdot s^3} \cdot \left\{1 - \left[1 - \frac{s^3}{(s + \delta)^3}\right] \cdot (1.45 \cdot A - 0.9 \cdot A^2 + 0.2 \cdot A^3)\right\}
\]

Where

\[
A = \frac{(b_1 + \sqrt{2 \cdot r \cdot \delta}) \cdot z}{D_{av}}
\]

K – Elastic Ring radial stiffness

The Precision of the formula is 1.5-5.0 %
A simulation for the system equipped with an Elastic ring (as described above) was performed.

In this case, the damping used is $\zeta=0.05$.

The damping of the system with an elastic ring is higher than the damping of the system without the elastic ring, because adding elements (such as an elastic ring) to a system increases the friction. This leads to more energy losses, and as a consequence to a higher damping.
The simulation does not take the gyroscopic effect into account, because it is of no interest to the subject of this study.

The frequency response is shown in slide 13 and the maximum support reaction in slide 14.

With the addition of the elastic ring, the critical rotation speed and the maximum support reaction become lower.
After we placed an elastic ring around the bearing, the stiffness of the support became nonlinear.

The nonlinearity of the support stiffness can be seen in the figure in slide 16.

In the first segment of the graph in slide 16, the support has the combined stiffness of the bearing and of the elastic ring.

In the second segment the support has the stiffness of the bearing only.

The transition between the stiffness of the bearing and the ring together to the stiffness of the bearing only occurs when the gap closes and the elastic ring stops being active.
Crossing the Resonant Frequency

Refer to the figure in slide 19. The rotor starts operating, supported by the ring with a low stiffness. The amplitude of the support reaction is not higher than the blue line: A-B-C.

At the point C, the gap closes, (the elastic rings becomes ineffective) and the amplitude of the support reaction goes down, until the pink line (D-D1), and the support stiffness is equal to the stiffness of the bearing without the elastic ring.
After the **point E**, the amplitude of the support reaction is not higher than the **line E-F-G-J (blue)**

In the **zones ABCD and EDHGFJ**, the simulation is a **simple linear study**.

In the **zones CDD1E** the stiffness is nonlinear and the simulation is much more complicated.
Magnitude of the Maximal Reaction as Rotation speed Function:

curve ABC ->

-> CDD1E zone->

-> curve EFGJ
Conclusion

If elastic rings, such as Allisson’s dampers are used, it is possible to get a rotor with very low support reaction when in resonance.