# A new approach to map prediction of centrifugal compressors

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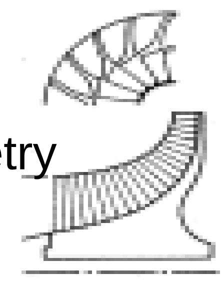
# The key message

- A reliable estimate of the achievable performance map of successful stages can be generated using non-dimensional information of the duty, with minimum information of the geometry.
  - Design point

$$\pi, \dot{V}, N, D_2, \eta_d \Rightarrow \eta_d, \lambda_d, \phi_d, M_{u2,d}, D_2$$

Design



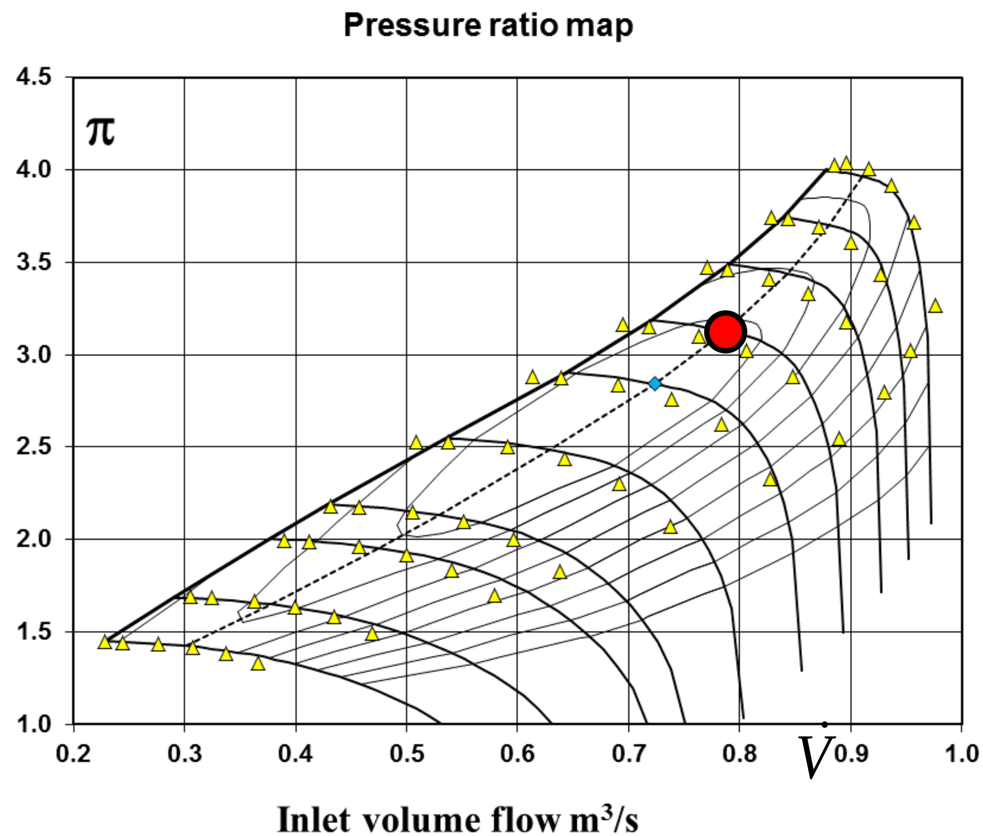
  - Geometry

Correlations, CFD or Tests

New approach

  - Performance map

$$\eta, \pi = f(\dot{V}, N)$$

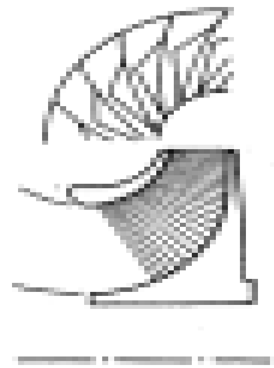
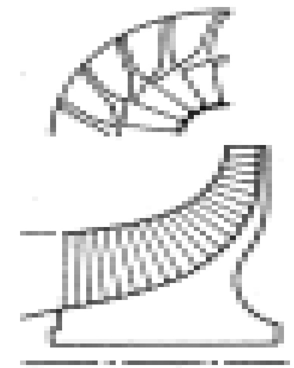


# How is this possible?

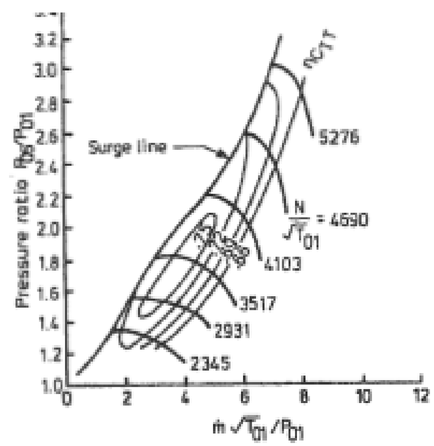
- The predicted map shows what should be achieved for a stage designed with these non-dimensional parameters – it does not apply to poor designs - which may have poor maps!
- Most stages are optimised with similar design rules so that good stages designed by different people have closely similar maps
  - Inlet optimised for minimum relative Mach number at the tip $M_{rel1}$
  - Limit to diffusion in shroud streamline: De Haller number $W_2/W_1 \sim 0.6$
  - Blade number selected on the basis of common loading criteria
  - Compromise between range and pressure rise give similar backsweep levels
  - Diffuser and impeller usually adapted for low incidence and good matching with throat areas selected for maximum flow requirements
- The method relies on the estimated efficiency and work at design and scales all other points from this.
- The method distinguishes between different types of stage.

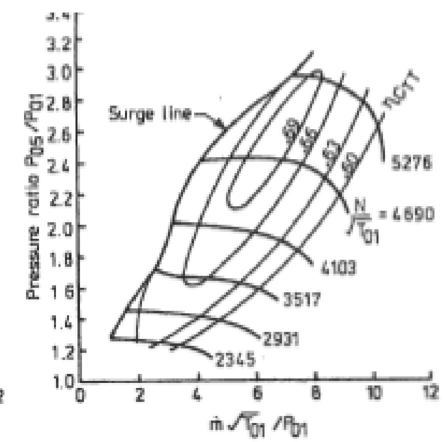
# Four different stage types

- Two types of impeller
  - Process compressor impeller
    - Radial inlet
    - Leading edge in inlet bend
    - Short shrouded impeller
    - Usually without splitter vanes
  - Inducer style impeller
    - Axial inlet
    - Long open impeller
    - Usually with splitter vanes
- Two types of diffuser
  - Vaneless diffusers
  - Vaned diffusers

Impeller: Process Inducer






Diffuser: Vaned Vaneless

- Different coefficients are selected for the four different types of stages

# Contents

# Key equations and parameters

- The method requires values of the key non-dimensional parameters of the stage at the peak efficiency point at its design speed:
  - Flow coefficient     Work coefficient     Efficiency     Tip-speed Mach number

$$\phi_d = \frac{\dot{V}}{u_2 D_2^2} \qquad \lambda_d = \frac{\Delta h_t}{u_2^2} \qquad \eta_d \qquad M_d = \frac{u_2}{a_{t1}} = \frac{u_2}{\sqrt{\gamma R T_{t1}}}$$

- The method generates stage characteristics for the individual stages with no further detailed information about the geometry, other than the impeller diameter and backsweep

$$\lambda, \eta = f(\phi, M, \ldots)$$

- These can be used with thermodynamic equations to predict the pressure ratio, and volume flow over a range of speeds

$$\pi = \left[1 + (\gamma - 1)\eta_s \lambda M^2\right]^{\gamma/(\gamma-1)} \qquad \dot{V} = \phi D_2^2 u_2 = \phi D_2^2 a_{t1} M$$

# Design of equations for model of stage characteristics

- Physical arguments have been used to select the most appropriate form of equations relating the non-dimensional performance variables
  - Equations were chosen so that geometry is not needed
  - Suitability of equations tested by comparison with test data
- Equations required
  - Efficiency
    - Variation of efficiency with flow along each speed line
    - Change in peak efficiency with speed
    - Flow coefficient at choke as a function of speed
    - Flow coefficient at peak efficiency as a function of speed
  - Work coefficient
    - Change in work coefficient with flow and speed
  - Surge line
    - Flow coefficient at surge at different speeds

# The aero-thermodynamic model

- Efficiency characteristics $\eta = f(\phi, M, \quad \phi_d, \lambda_d, \eta_d, M_d, \quad A, B, C, D, \ldots)$
  - Dependent variables
    - polytropic efficiency and work coefficient $\eta, \lambda$
  - Independent variables
    - flow coefficient and tip-speed Mach number $\phi, M$
  - Non-dimensional parameters at design point
    - Selected by the user $\phi_d, \lambda_d, \eta_d, M_d$
  - Variable coefficients and fixed constants
    - Selected to match historical test data $A, B, C, D, \ldots$
- Work characteristics
  - Derived from the 1D Euler equation (see later)

$$\lambda = f(\phi_{t1}, M_{u2}) = \left(1 + \frac{k}{\phi_{t1}}\right)\left(1 - \frac{c_s}{u_2} + \frac{\phi_{t1}}{\left[1 + (\gamma - 1)\gamma_{imp}\lambda M_{u2}^2\right]^{\frac{1}{n_{imp}-1}}} \frac{D_2 \tan \beta_2^{'}}{b_2 \pi}\right)$$

# Anatomy of a performance map

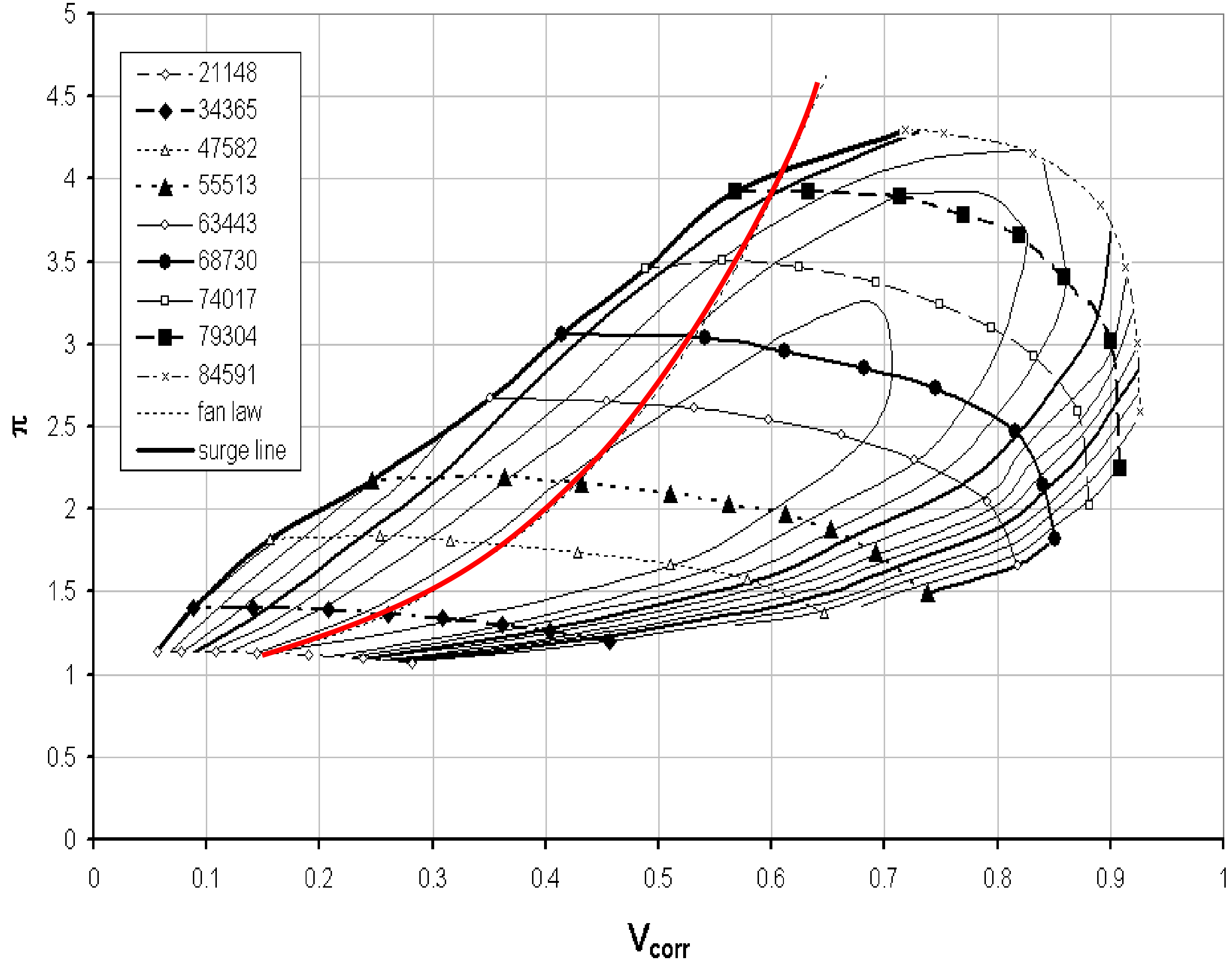
- Map as measured

$$\pi, \eta = f(\dot{V}, N)$$

  - Pressure ratio versus volume flow on different speed -lines
  - Contours of efficiency
  - Surge line
  - Fan law

$$\Delta p \propto N^2$$

$$\dot{V} \propto N$$



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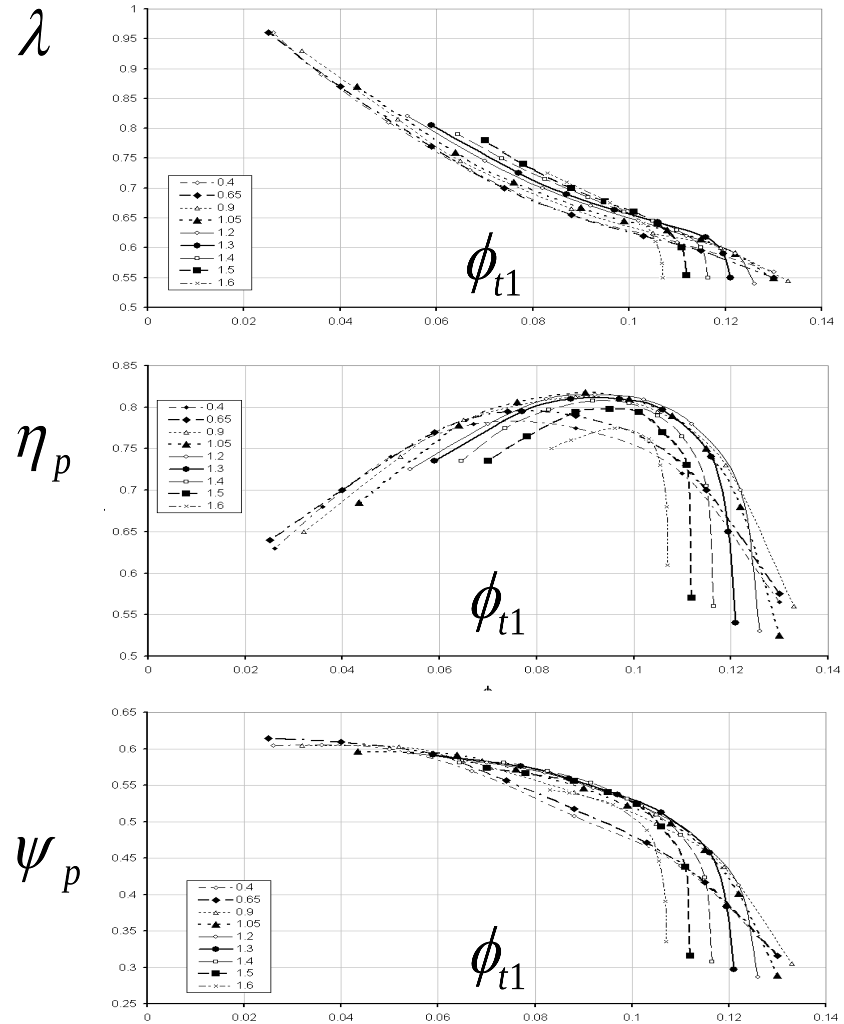
# Conversion of map to stage characteristic curves

- Work coefficient, polytropic efficiency and pressure coefficient versus inlet flow coefficient
  - Parameter of the speed-lines
    - Tip-speed Mach number

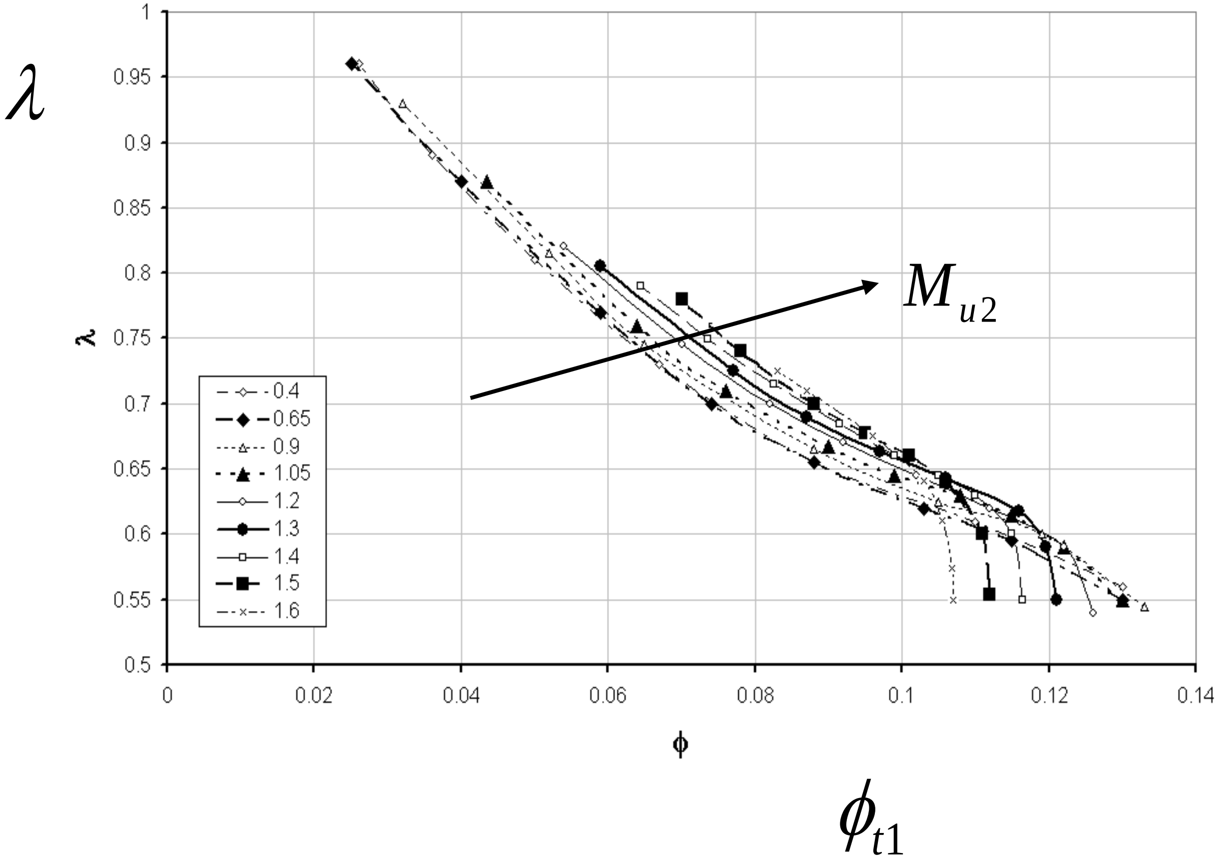
$$\lambda, \eta_p, \psi_p = f(\phi_{t1}, M_{u2})$$

- Effect of tip-speed Mach number
    - Density variation across impeller
    - Choking at impeller or diffuser inlet
- New approach is based on $\lambda$ and $\eta_p$ as
  - $\psi_p = \lambda \eta_p$
  - Euler equation is available for work
  - Efficiency equations for losses



# Anatomy of work input characteristic

- Work input coefficient versus inlet flow coefficient

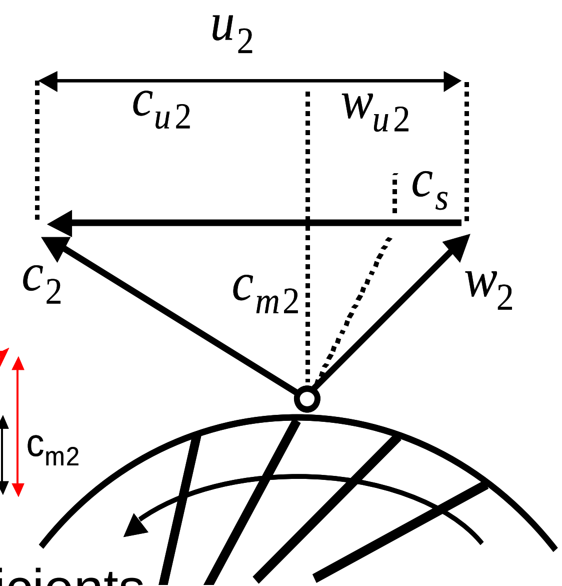



$$\lambda = \frac{\Delta h_t}{u_2^2}$$
$$= f(\phi_{t1}, M_{u2})$$

# Model of work transfer based on Euler equation

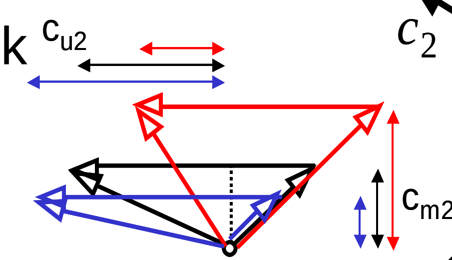
- Work input coefficient versus impeller outlet flow coefficient
  - Euler equation for work done on gas

$$\lambda_{Euler} = \frac{c_{u2}}{u_2} = 1 - \frac{c_s}{u_2} + \phi_2 \tan\beta_2'$$




- Modification for disc friction work

$$\lambda = \left(1 + \frac{k}{\phi_{t1}}\right)\lambda_{Euler}, \quad k \approx 0.004$$



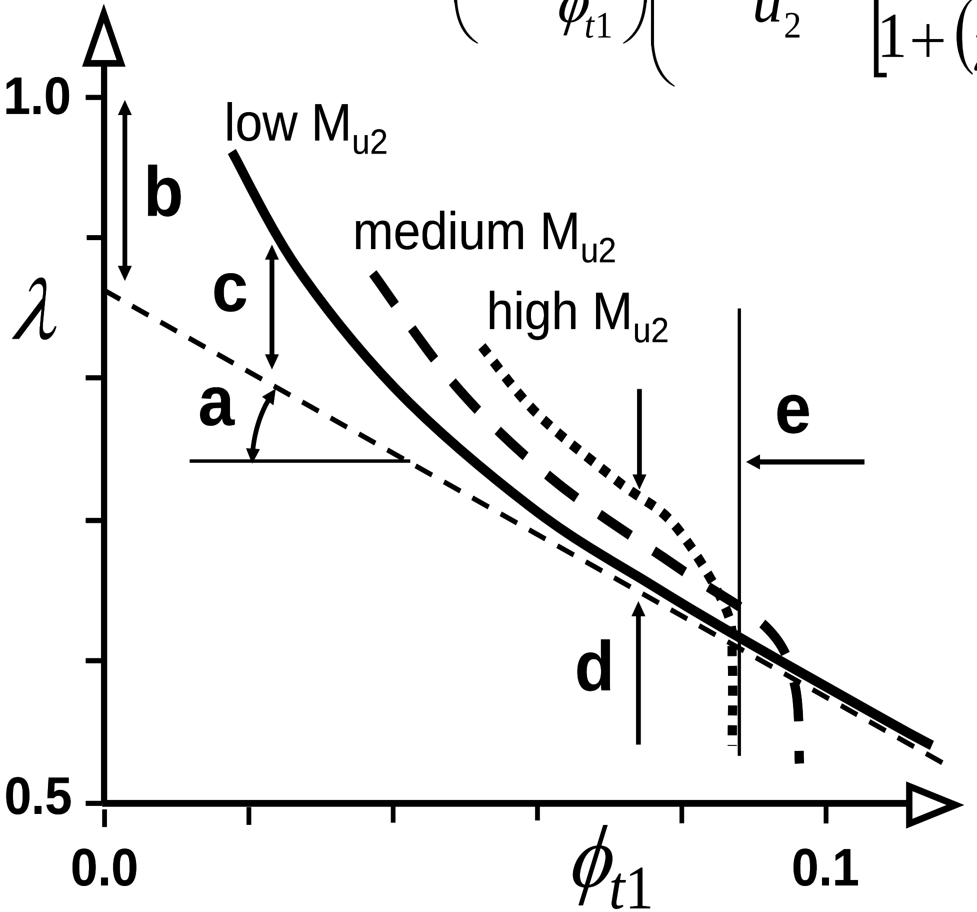

- Relationship between inlet and outlet flow coefficients

$$\dot{m} = \pi b_2 D_2 \rho_2 u_2 \phi_2 = D_2^2 \rho_{t1} u_2 \phi_{t1}$$

$$\phi_2 = \frac{c_{m2}}{u_2} = \phi_{t1} \frac{1}{\pi} \frac{D_2}{b_2} \frac{\rho_{t1}}{\rho_2} \qquad \frac{\rho_2}{\rho_{t1}} = \left[1 + (\gamma - 1)\gamma_{imp} \lambda M_{u2}^2\right]^{\frac{1}{n_{imp}-1}}$$

# Shape of work coefficient characteristic

$$\lambda = f(\phi_{t1}, M_{u2}) = \left(1+\frac{k}{\phi_{t1}}\right)\left(1-\frac{c_s}{u_2}+\frac{\phi_{t1}}{\left[1+(\gamma-1)\gamma_{imp}\lambda M_{u2}^2\right]^{\frac{1}{n_{imp}-1}}}\frac{D_2\tan\beta_2'}{b_2\pi}\right)$$

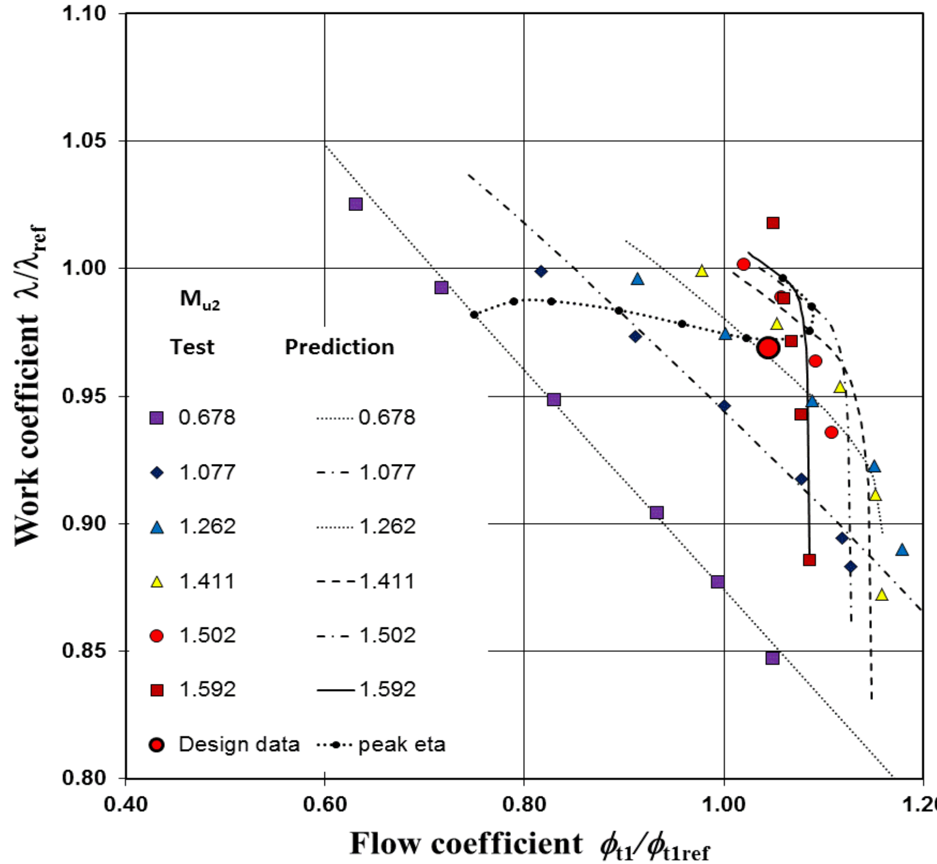



- Five main effects
  - a) Back-sweep
  - b) Slip factor
  - c) Disc friction
  - d) Density change between inlet and outlet of impeller
  - e) Choking in impeller inlet

# Validation of work equation

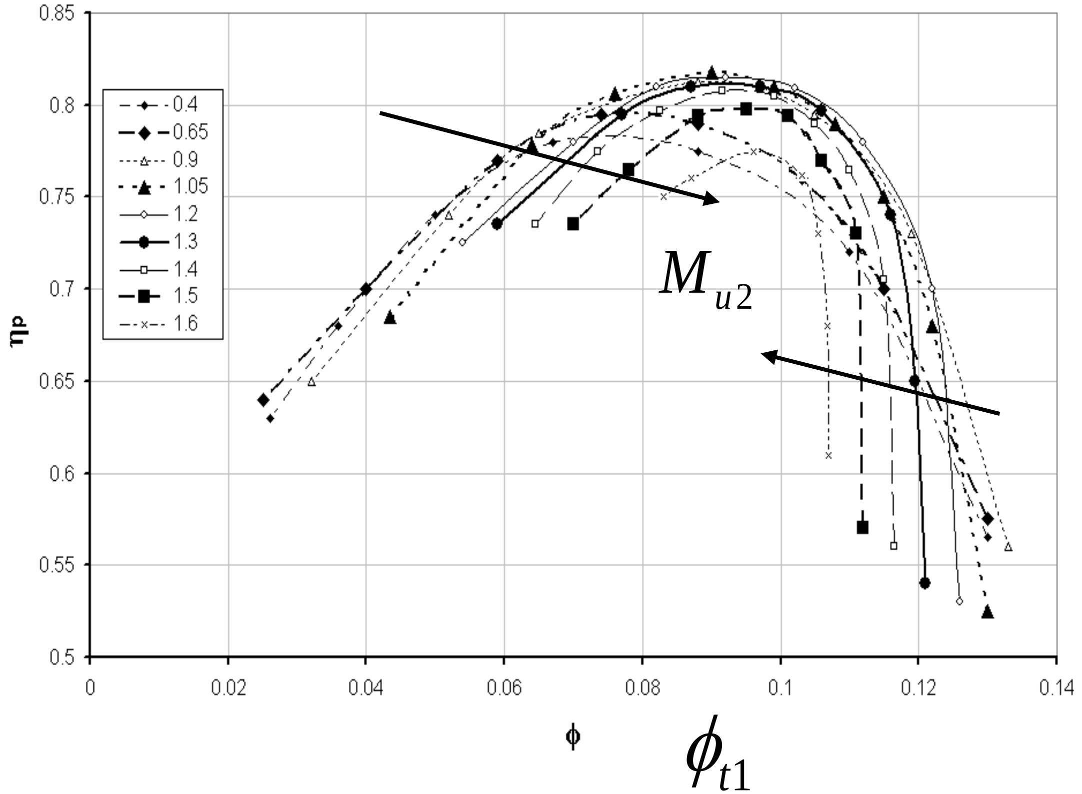
$$\lambda = f(\phi_{t1}, M_{u2}) = \left(1 + \frac{k}{\phi_{t1}}\right)\left(1 - \frac{c_s}{u_2} + \frac{\phi_{t1}}{\left[1 + (\gamma - 1)\gamma_{imp}\lambda M_{u2}^2\right]^{\frac{1}{n_{imp}-1}}}\frac{D_2 \tan\beta_2^{'}}{b_2\pi}\right)$$

- The geometry parameter
$$\frac{D_2 \tan\beta_2^{'}}{b_2\pi}$$
is adjusted to give the design value work input at the design point
- Design point
- The work at design is specified and the equation above is only used to predict the variation of work with flow and speed from the design point



# Anatomy of efficiency characteristic

- Non dimensional efficiency versus flow coefficient
- Increase of tip-speed Mach number $M_{u2}$
  - Causes peak efficiency to increase then to decrease
  - Causes a shift in the location of peak efficiency to higher flow coefficients
  - Causes characteristics to change shape and become narrower
  - Causes choke to move closer to peak efficiency



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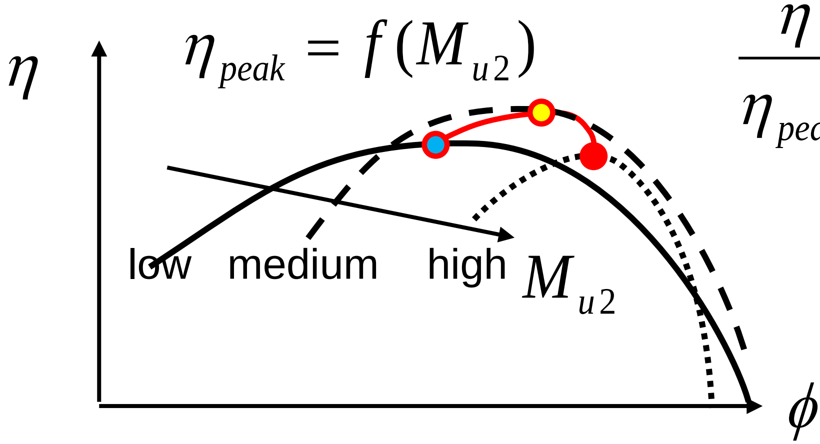
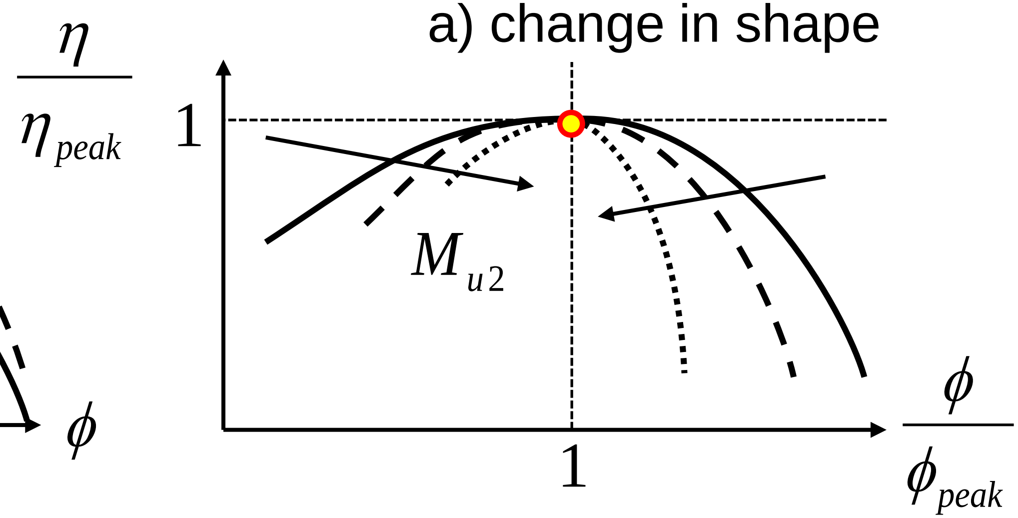
# Anatomy of efficiency variation

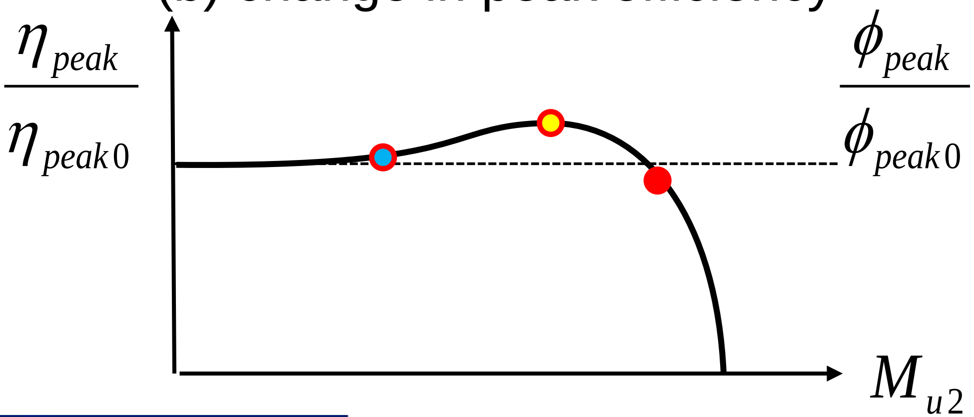
- Normalisation of efficiency versus flow characteristic shows 3 effects



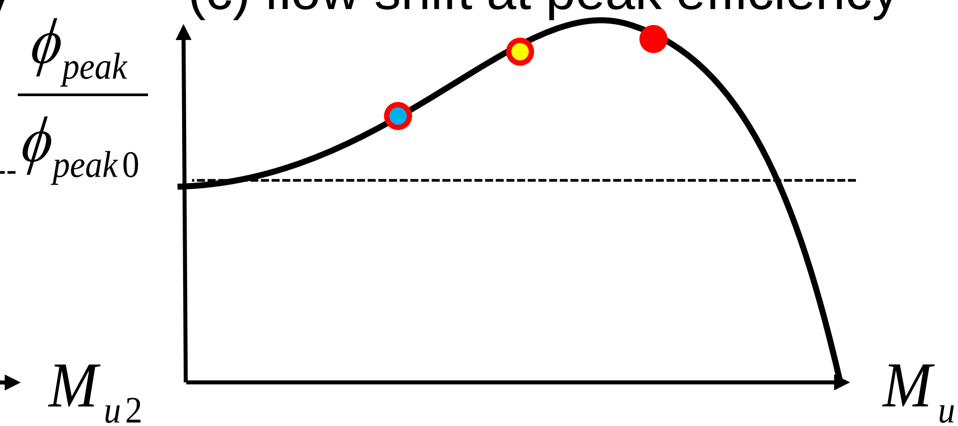

a) change in shape




(b) change in peak efficiency

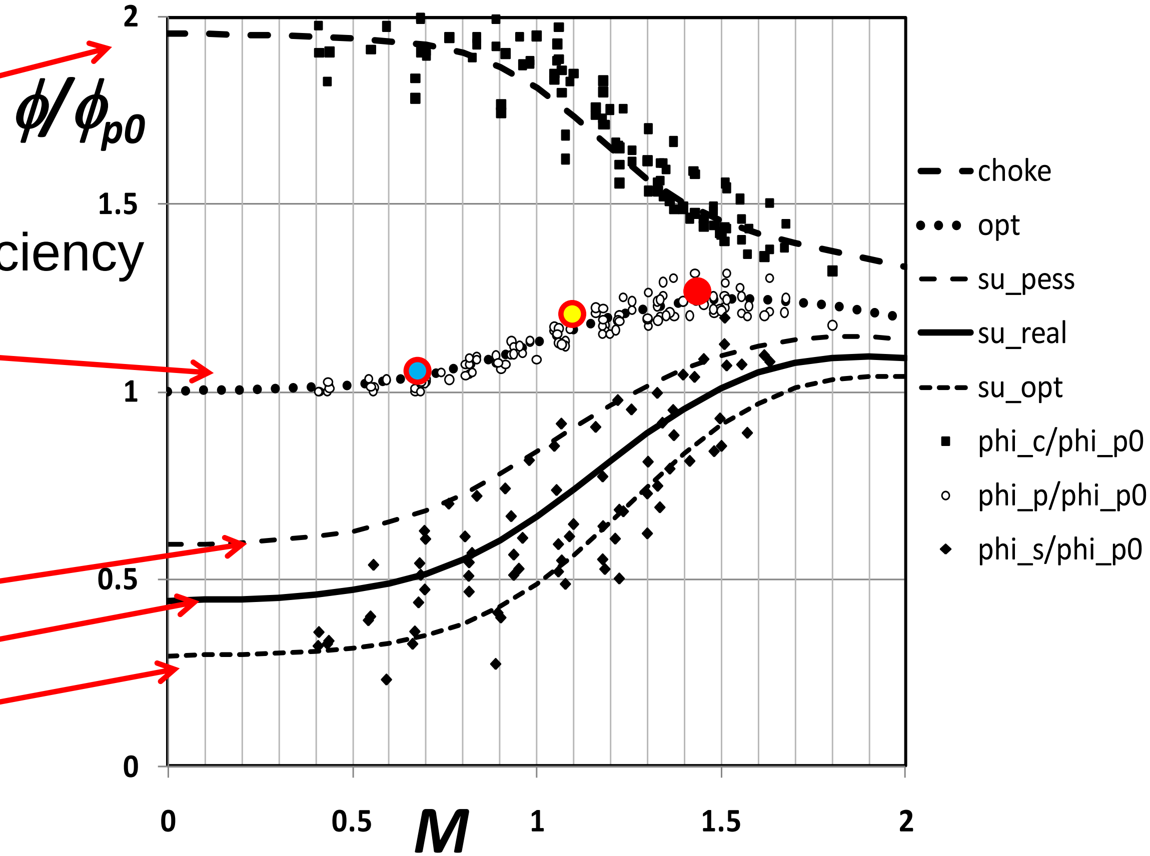



(c) flow shift at peak efficiency



# Flow coefficient versus Mach number envelope for a turbocharger stage with a vaneless diffuser

- Flow shift at choke
  $\phi_c / \phi_{p0} = f(M)$
- Flow shift at peak efficiency
  $\phi_p / \phi_{p0} = f(M)$
- Flow shift at surge
  $\phi_s / \phi_{p0} = f(M)$
  - Pessimistic
  - Realistic
  - Optimistic




- Validation data for 15 different turbochargers with vaneless diffusers

# Model for flow coefficient at choke

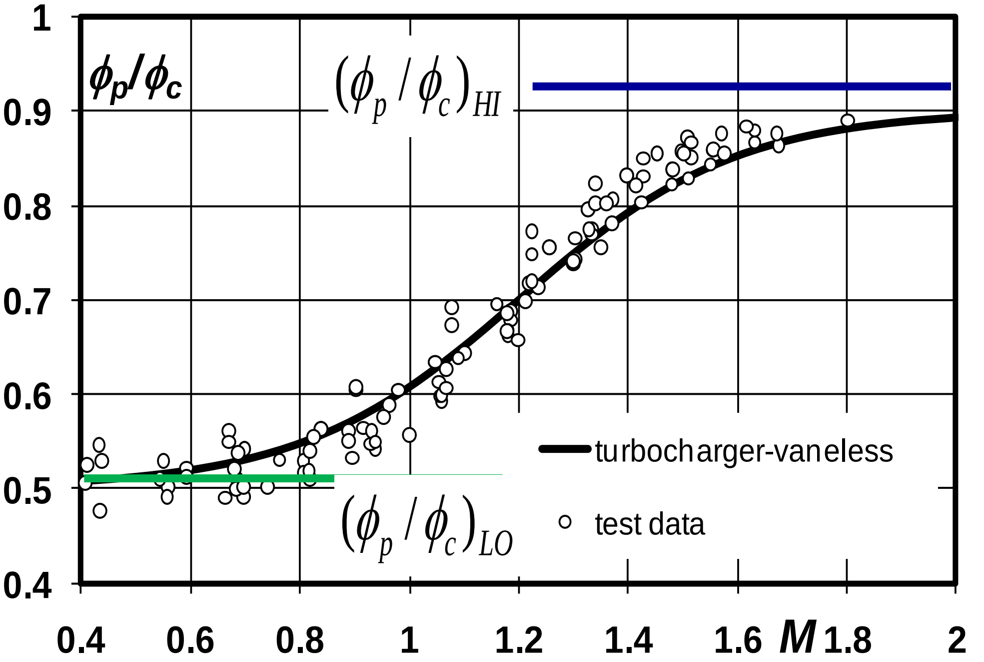
$$\frac{\phi_p}{\phi_c} = f(M_{u2})$$

- Range decreases with speed but is constant at both high (HI) and low (LO) speeds, for example for vaneless stages

$$\left(\frac{\phi_p}{\phi_c}\right)_{LO} = 0.5, \quad \left(\frac{\phi_p}{\phi_c}\right)_{HI} = 0.92$$

- Test data shows an s-shape between the two asymptotes
- Equation selected to model this

$$\frac{\phi_p}{\phi_c} = (1-P)\left(\frac{\phi_p}{\phi_c}\right)_{LO} + P\left(\frac{\phi_p}{\phi_c}\right)_{HI}$$




$$P = \frac{1}{1+e^{-t}}, \quad t = (M_{u2} - B)(AM_{u2} + C), \quad P = 0.5 \ at \ t = 0 \ and \ M_{u2} = B$$

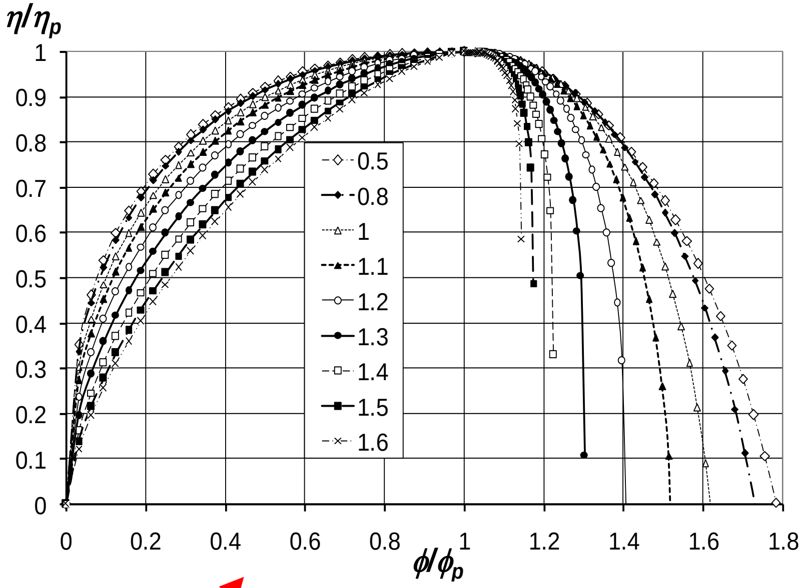
– The blending function P is known as the logistic function

# Model for variation of efficiency at low flow $\phi < \phi_p$

- Equations for flows below peak efficiency
  - Similar to the equation for an ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \quad \frac{x}{a} = \left[1 - \left(\frac{y}{b}\right)^2\right]^{1/2}$$

  - Exponent varies to give different shapes
    - D = 2 would give an elliptical equation
    - Typically $D_{LO}$ = 2.1 and $D_{HI}$ = 1.7
  - S-shaped blending function P, as given before

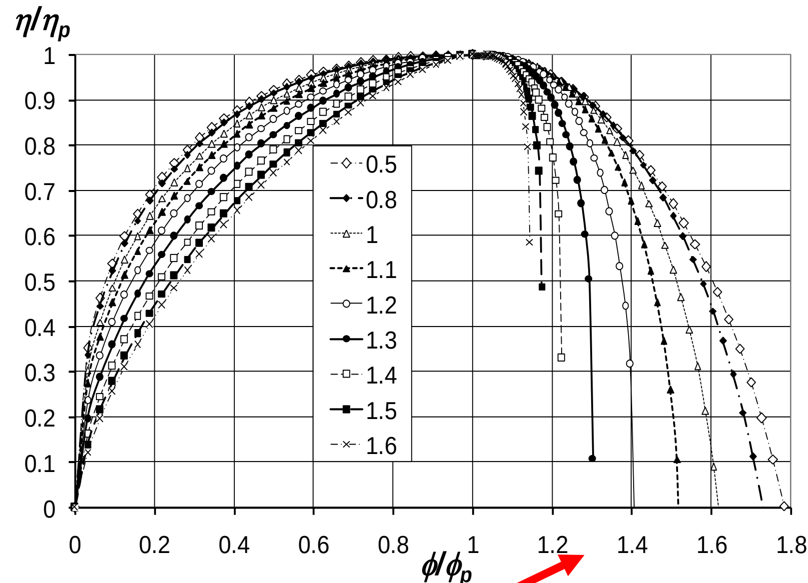



$$\phi < \phi_p, \quad \frac{\eta}{\eta_p} = \left[1 - \left(1 - \frac{\phi/\phi_c}{\phi_p/\phi_c}\right)^D\right]^{1/D}$$

$$D = D_{LO}(1 - P) + D_{HI} P$$

# Model for variation of efficiency at high flow $\phi > \phi_p$

- Equations for flows above peak efficiency
  - Exponent varies to modify shape of curves
    - H = 2 would give an elliptical equation
    - H > 2 gives a more flat-topped curve typical of transonic stages
    - $H_{LO}$ = 2 and $H_{HI}$ = 3.5
  - Efficiency ratio adjusted so that efficiency ratio at choke is not zero but given by 1-G
    - $G_{LO}$ = 2 and $G_{HI}$ = 0.7
  - Blending function P as given before



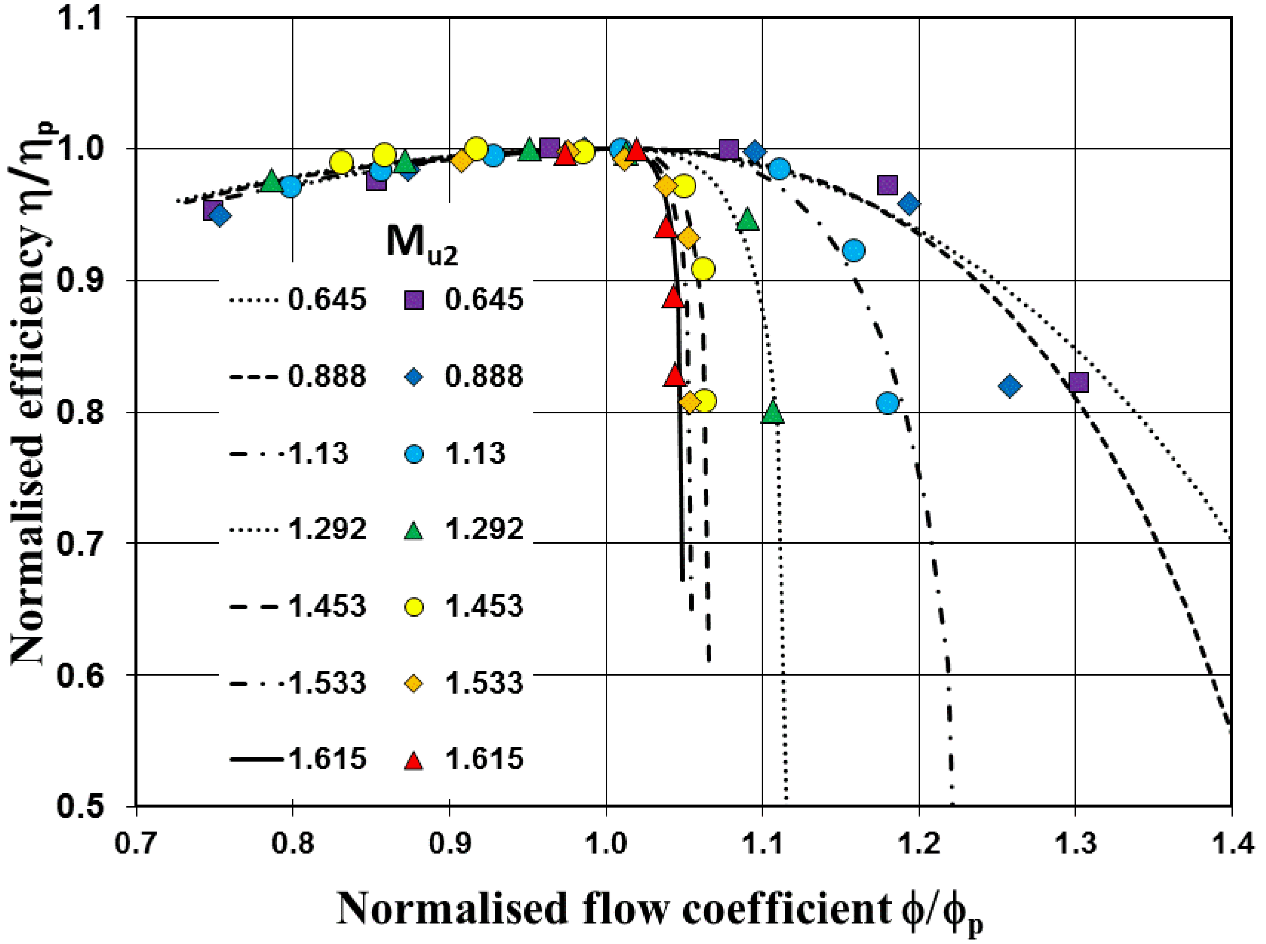

$$\phi > \phi_p, \quad \frac{\eta}{\eta_p} = (1-G) + G\left[1-\left(\frac{\dfrac{\phi}{\phi_c} - \dfrac{\phi_p}{\phi_c}}{1-\dfrac{\phi_p}{\phi_c}}\right)^H\right]^{1/H}$$

$$G = G_{LO}(1-P) + G_{HI}P$$

$$H = H_{LO}(1-P) + H_{HI}P$$

# Calibration of shape parameters A, B, C, D, E, F, G and H

- Tests on over 30 vaned diffuser stages used for selecting coefficients
- Additional cases used to validate the approach
- The case shown was not used to establish the coefficients



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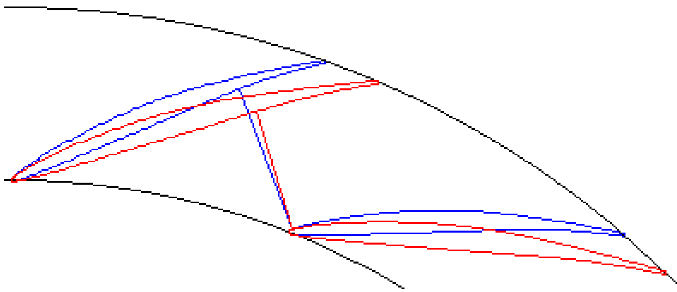
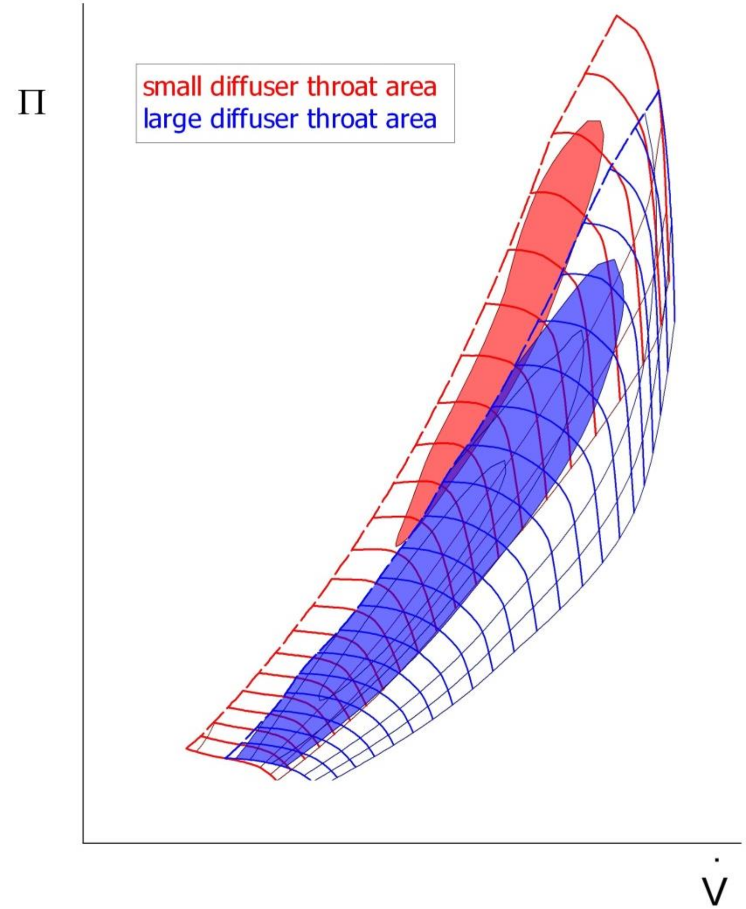
# Effect of matching of a vaned diffuser

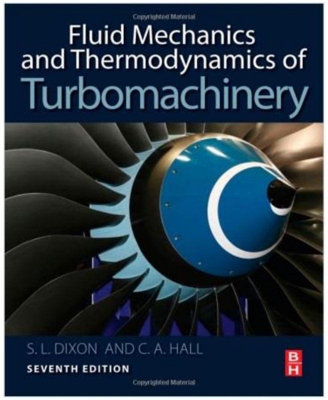


- A change in diffuser throat area causes a large change in the map
  - In this case the impeller and flow channel are unchanged
- This is a common procedure to adapt compressors to different requirements
- The smaller diffuser throat leads to
  - a higher pressure ratio at high speeds
  - higher efficiency at higher speeds
  - a slightly lower flow at a given speed
  - less steep speed lines at high speed
- Surely we need a geometry parameter to model this effect? No!



# Derivation of a 1D matching equation

- 1D equations for maximum flow per unit area (Dixon and Hall, 7th ed.)
  - Impeller $$\frac{\dot{m}}{A_i^*} = \rho_{t1} a_{t1} \left[ \frac{2 + (\gamma - 1) u^2 / a_{t1}^2}{\gamma + 1} \right]^{\frac{(\gamma+1)}{2(\gamma-1)}}$$
  - Diffuser $$\frac{\dot{m}}{A_d^*} = \rho_{t2} a_{t2} \left[ \frac{2}{\gamma + 1} \right]^{\frac{(\gamma+1)}{2(\gamma-1)}}$$




- Dimensionless form $$\phi_{t1} = \frac{\dot{V}}{u_2 D_2^2} \qquad \lambda = \frac{\Delta h_t}{u_2^2} \qquad M_{u2} = \frac{u_2}{a_{t1}}$$
  - Impeller $$\phi_{t1}^i = \frac{1}{M_{u2}} \frac{A_i^*}{D_2^2} \left[ \frac{2 + (\gamma - 1)(D_1 / D_2)^2 M_{u2}^2}{\gamma + 1} \right]^{\frac{(\gamma+1)}{2(\gamma-1)}}$$
  - Diffuser $$\phi_{t1}^d = \frac{1}{M_{u2}} \frac{A_d^*}{D_2^2} \left[ \frac{1 + (\gamma - 1) \lambda M_{u2}^2}{1} \right]^{\frac{(n+1)}{2(n-1)}} \left[ \frac{2}{\gamma + 1} \right]^{\frac{(\gamma+1)}{2(\gamma-1)}}$$

# Optimum matching

- We assume that the impeller and diffuser are well matched when both choke the same inlet flow coefficient as this gives the widest range possible.

$$\phi_{t1}^{d} = \phi_{t1}^{i} \qquad \frac{A_d^*}{A_i^*} = \frac{\left[1 + (1/2)(\gamma - 1)(D_1/D_2)^2 M_{u2}^2\right]^{\frac{(\gamma+1)}{2(\gamma-1)}}}{\left[1 + (\gamma - 1)\lambda M_{u2}^2\right]^{\frac{(n+1)}{2(n-1)}}}$$

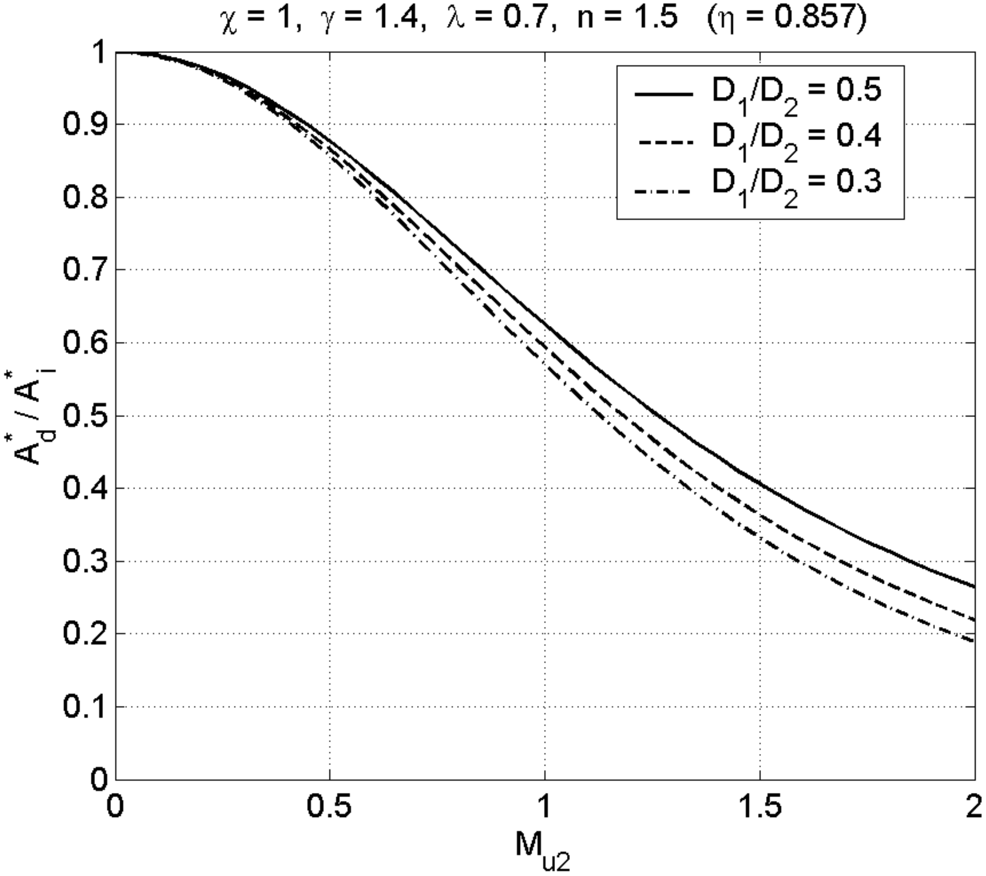
- For given values of $M_{u2}$, $\gamma$, $D_1/D_2$, $n$ and $\lambda$ we can calculate the required area of the diffuser throat relative to that of the impeller throat $A_d/A_i$ for optimum matching.

- Alternatively, for a given area ratio $A_d/A_i$, $\gamma$, $D_1/D_2$, $n$ and $\lambda$ we can calculate the tip speed Mach number $M_{u2}$ that would correspond to optimum matching of impeller and diffuser, which would normally be the design value.

# Variation of the ratio of diffuser to impeller throat area

$$\frac{A_d^*}{A_i^*} = \frac{\left[1+(1/2)(\gamma-1)(D_1/D_2)^2 M_{u2}^2\right]^{\frac{(\gamma+1)}{2(\gamma-1)}}}{\left[1+(\gamma-1)\lambda M_{u2}^2\right]^{\frac{(n+1)}{2(n-1)}}}$$

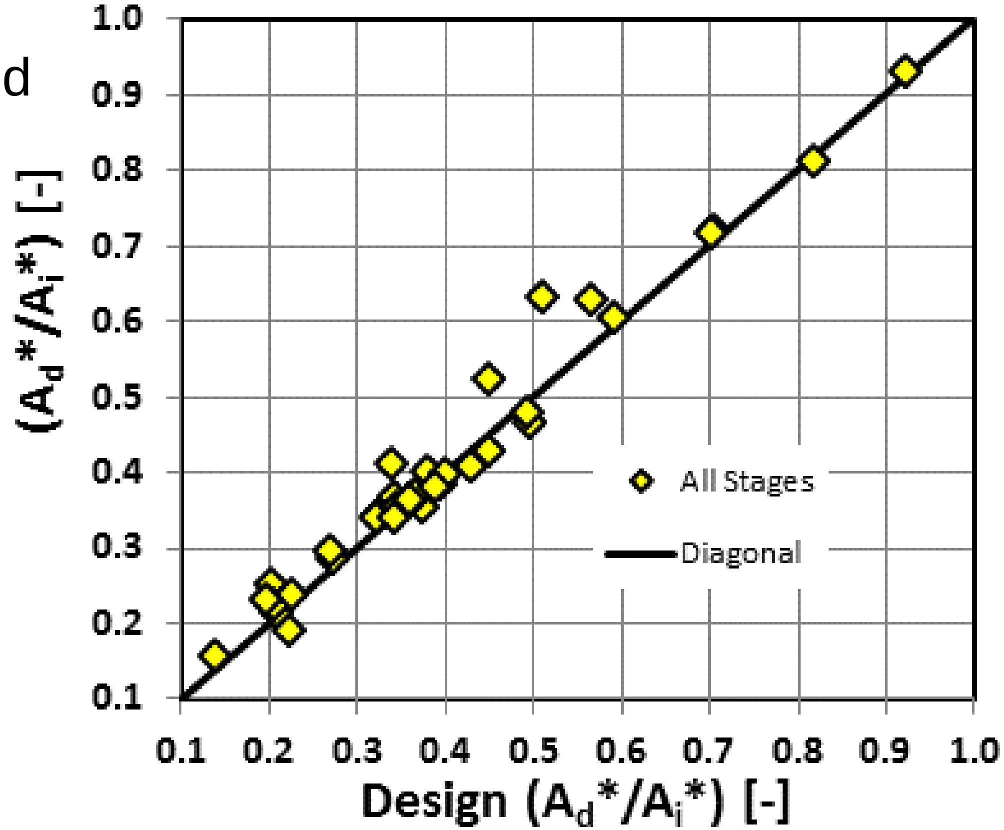
- The diffuser requires a smaller throat area as the tip-speed Mach number increases
- Lower impeller inlet diameter $D_1/D_2$ also reduces the diffuser throat area
- Stages with a high work coefficient (less back-sweep) require a smaller diffuser
- Diffusers following an impeller with a higher efficiency also require a smaller diffuser.



# Validation with design data from many sources

$$\frac{A_d^*}{A_i^*} = \frac{\left[1+(1/2)(\gamma-1)(D_1/D_2)^2 M_{u2}^2\right]^{\frac{(\gamma+1)}{2(\gamma-1)}}}{\left[1+(\gamma-1)\lambda M_{u2}^2\right]^{\frac{(n+1)}{2(n-1)}}}$$

- X-axis is the actual design throat area ratio of each stage
- Y-axis is the area ratio predicted at the design Mach number
- Design data covers
  - Pressure ratio: 1.2 to 12
  - Different impeller styles (open, shrouded, splitters)
  - Different diffuser styles (wedge, aerofoil, circular arc)
  - Different design philosophies
- Sources of data in given in the acknowledgements of Rusch and Casey (2014)



# New understanding from the matching equation

- There is no such thing as a mismatched diffuser!
  - It will always become matched at a different tip-speed.
- The 1D matching equation estimates the required relative throat areas $A_d/A_i$

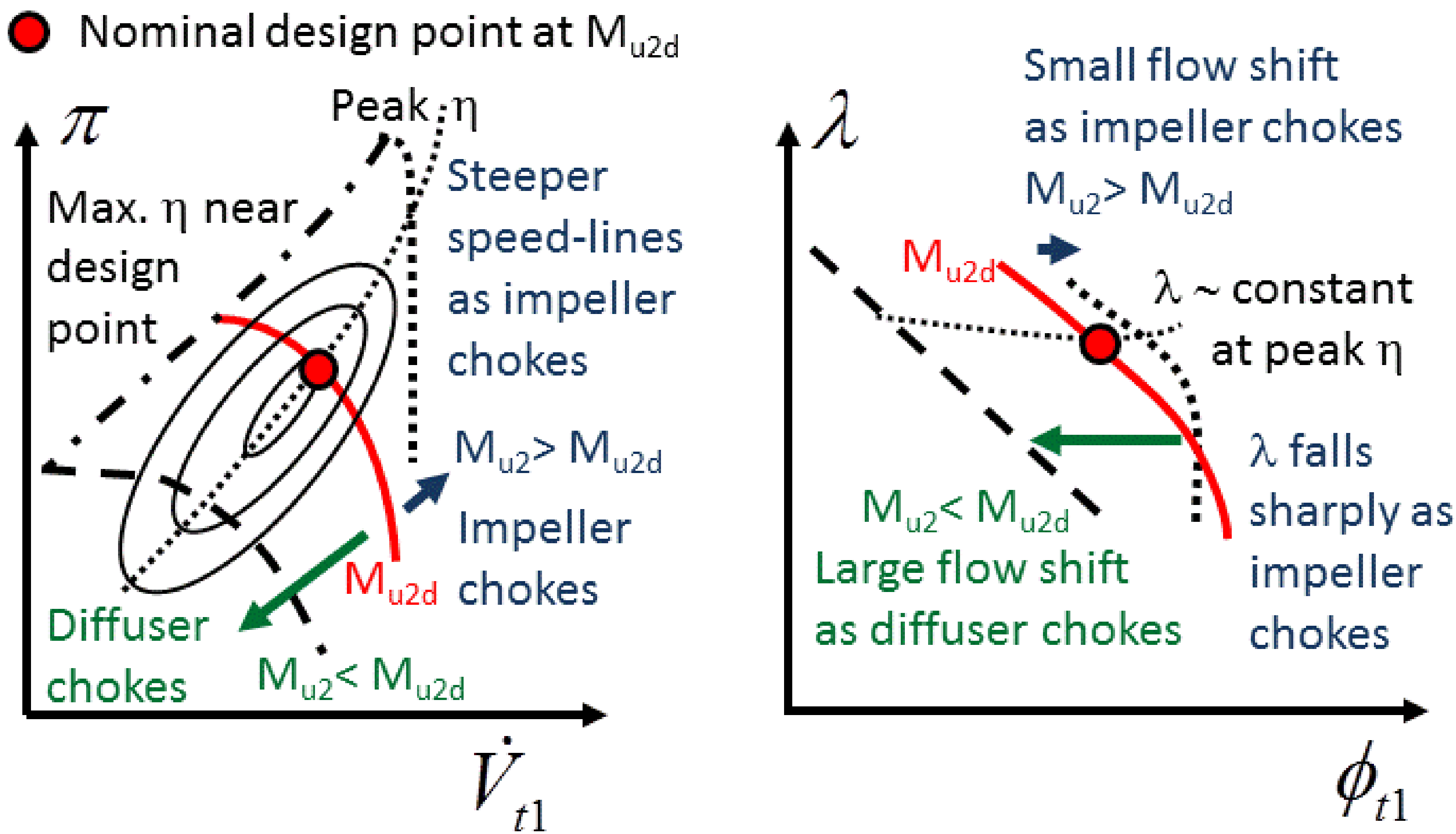
$$\frac{A_d^{\,*}}{A_i^{\,*}} = \frac{\left[1+(1/2)(\gamma-1)(D_1/D_2)^2 M_{u2}^{\,2}\right]^{\frac{(\gamma+1)}{2(\gamma-1)}}}{\left[1+(\gamma-1)\lambda M_{u2}^{\,2}\right]^{\frac{(n+1)}{2(n-1)}}}$$

  - The design tip-speed Mach number $M_{u2,d}$ can replace the relative throat areas $A_d/A_i$ as a geometry parameter in the equations
- If the throat area ratio $A_d/A_i$ is subsequently changed then the diffuser and the impeller become optimally matched at a different speed, so we have a new design tip-speed Mach number, $M_{u2,d}$

$$\frac{A_d^{\,*}}{A_i^{\,*}} = \frac{\left[1+(1/2)(\gamma-1)(D_1/D_2)^2 M_{u2}^{\,2}\right]^{\frac{(\gamma+1)}{2(\gamma-1)}}}{\left[1+(\gamma-1)\lambda M_{u2}^{\,2}\right]^{\frac{(n+1)}{2(n-1)}}} \quad \Rightarrow \quad M_{u2,d} = f\left(\frac{A_d^{\,*}}{A_i^{\,*}}\right)$$

# Summary of matching effects with vaned diffusers

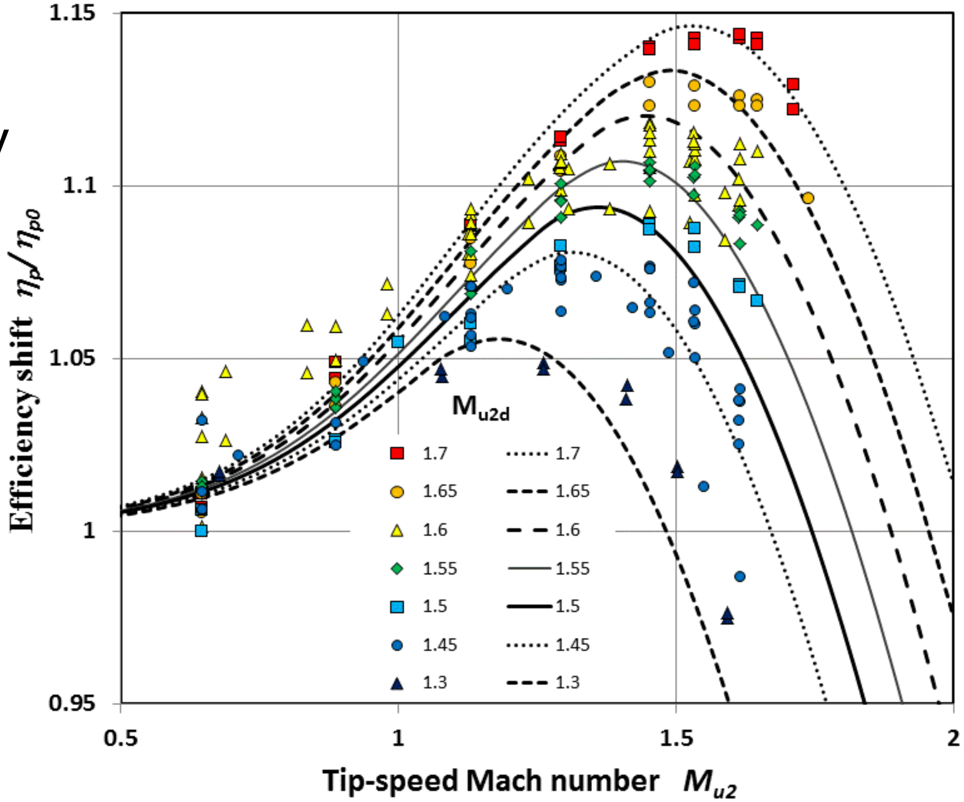
- Matching effects can be explained with the machine Mach number at which both components choke simultaneously, $M_{u2d}$



# Effect of diffuser matching on the peak efficiency

$$\frac{\eta_p}{\eta_{p0}} = f(M_{u2}, M_{u2d})$$

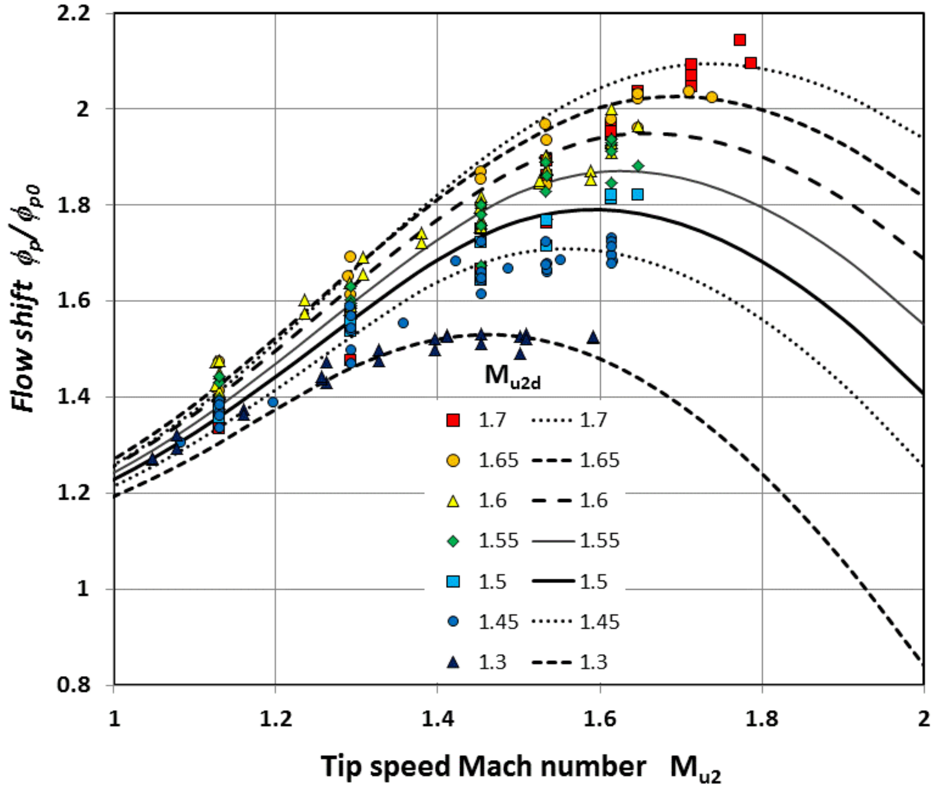
- The variation of the peak efficiency with speed depends on matching:
  - Efficiency is best close to the nominal design Mach number which has the best matching
  - Efficiency is poor at very low speeds due to poor matching as the diffuser is too small
  - Efficiency decreases at higher Mach numbers due to high-speed losses and poor matching with a diffuser that is too large



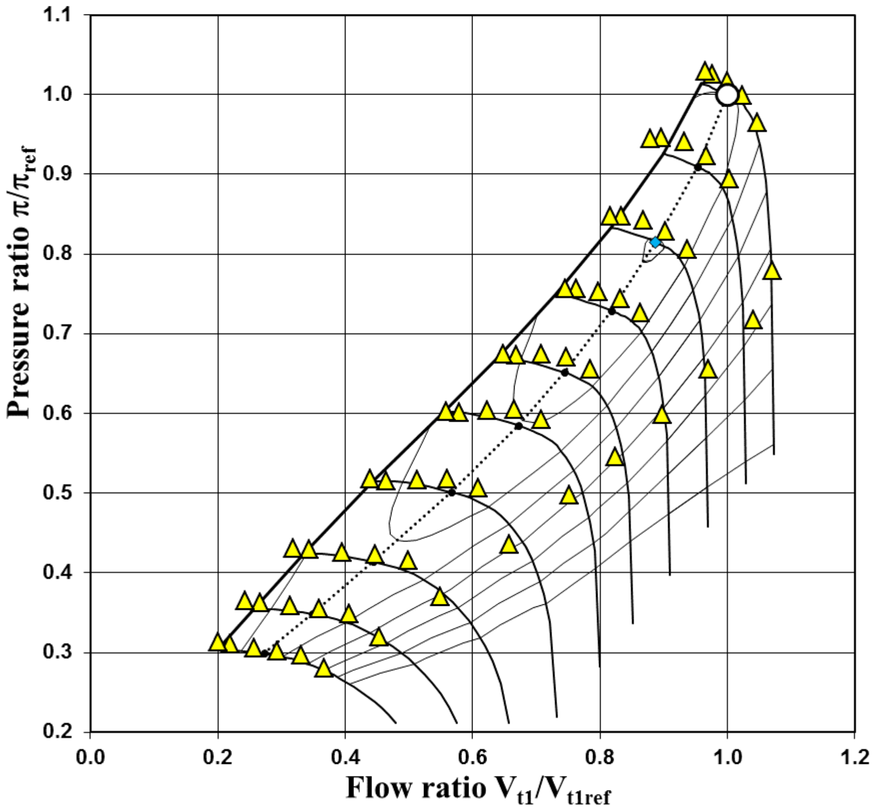
# Effect of diffuser matching on flow shift at peak efficiency

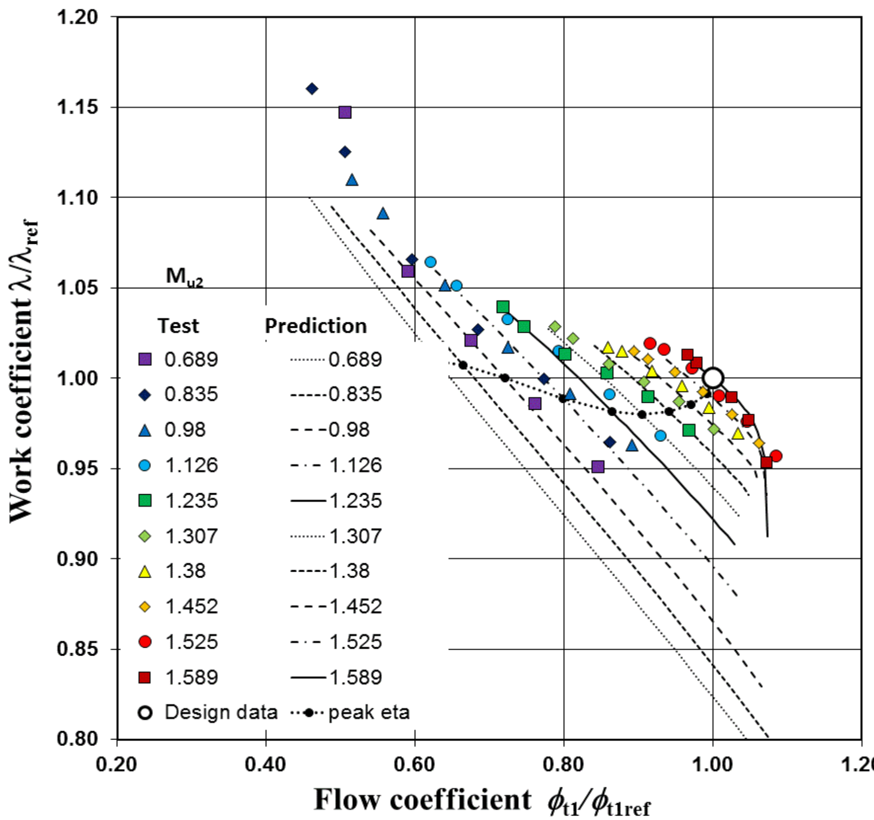
$$\frac{\phi_{t1p}}{\phi_{t1p0}} = f(M_{u2}, M_{u2d})$$

- The variation of the flow coefficient at peak efficiency with speed also depends on matching:
  - If the diffuser has a small diffuser to impeller throat area ratio it is matched at a high $M_{u2d}$ value
  - The diffuser then acts as a choked nozzle at low speeds and causes a very large reduction in the flow coefficient at low speed



# Prediction of a map for a stage with a small diffuser

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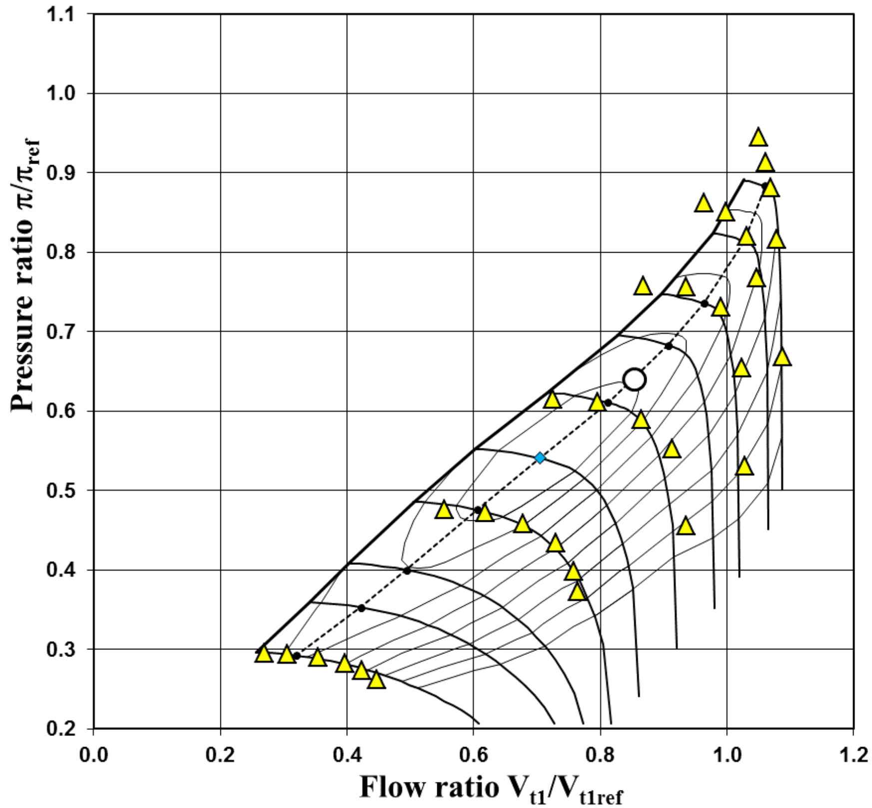
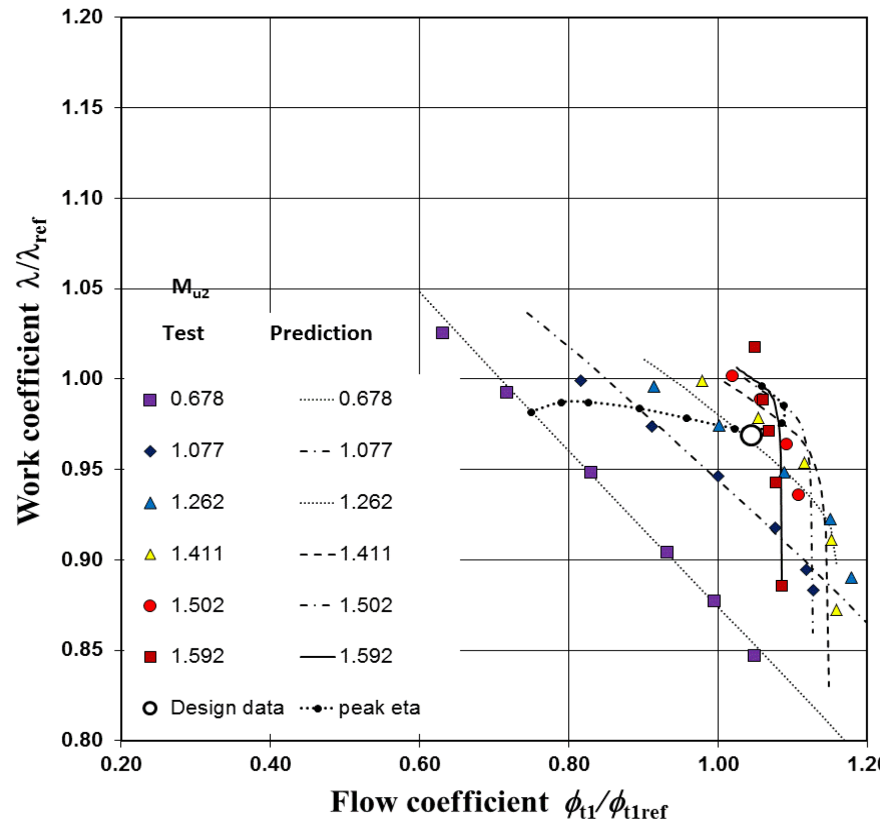
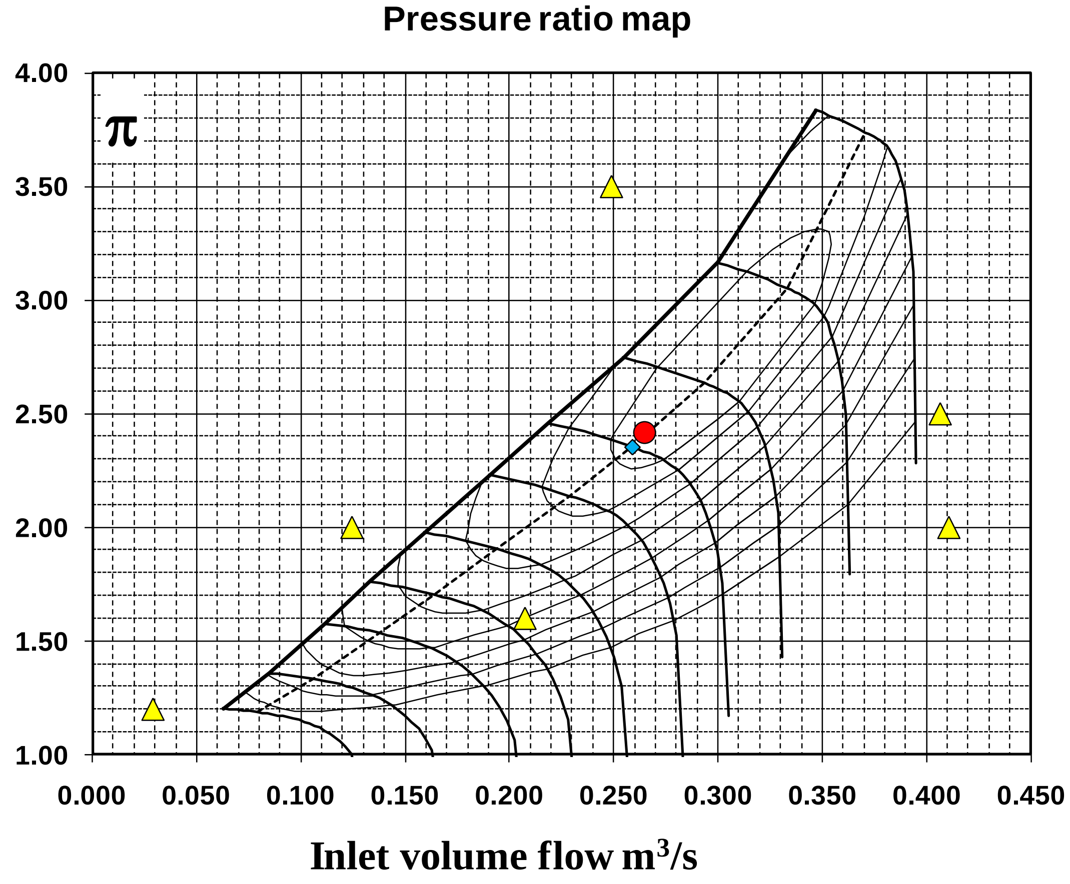
# Prediction of a map of same stage with a large diffuser



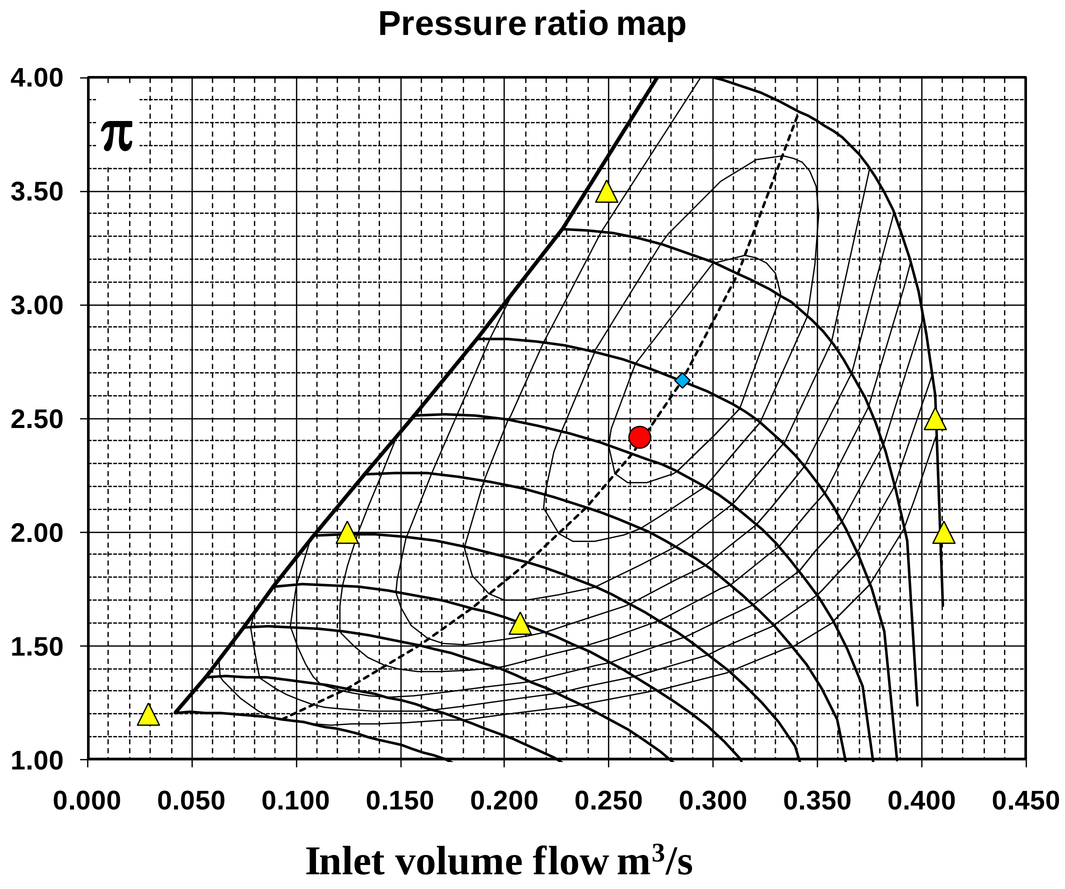
# Application in preliminary design and procurement (1)

- How difficult will it be to achieve the technical objectives?
- Calculation with a vaned diffuser using the mean coefficients and a realistic surge line
- Design point
  - Information at this point defines the whole map
- Other required operating points
  - Surge line will not be achieved
  - High speed choke is hard to achieve
  - Point at low pressure ratio has poor efficiency



# Application in preliminary design and procurement (2)

- We need to change the objectives or increase the range (Map Width Enhancement with inlet recirculation or a vaneless diffuser?)
- Calculation using a vaneless diffuser and standard coefficients
- Design point
  - Information at this point defines the whole map
- Other required operating points
  - Surge line is now just achievable
  - Choke is just OK
  - Better efficiency at low-speed point



# Summary of the new approach

- The method provides a simple, rapid and reliable way of estimating the achievable performance maps of well-designed centrifugal compressors at an early stage in the design process.
- The user specifies a few key non-dimensional parameters related to the compressor aerodynamic duty and from this single point an achievable performance map over the whole speed range is estimated.
- Only minimum information of the geometry of the stage is required.
- The method makes use of simple models for the stage characteristics that give the variation of efficiency and of work as a function of flow for varying tip-speed Mach numbers away from the specified design point.
- It also makes use of many empirical coefficients that are different for different types of stages but have been adjusted to match the measured performance of a wide range of successful stages.
- It is an extremely useful tool, especially in the preliminary design and procurement phases of a new design.

# References

- Technical publications with more information are available

  - Casey, M.V., and Robinson, C.J., (2006), “A guide to turbocharger compressor characteristics”, published in “Dieselmotorentechnik”, 10th Symposium, 30-31 March, 2006, Ostfildern, Ed. M. Bargende, , TAE Esslingen, ISBN 3-924813-65-5
  - Casey, M., and Robinson, C.J., (2013), “A Method to Estimate the Performance Map of a Centrifugal Compressor Stage”, ASME Journal of Turbomachinery, March 2013; Volume 135 (2): 021034 (10 pages); doi: 10.1115/1.4006590
  - Casey, M., Rusch, D., (2014), “The Matching of a Vaned Diffuser With a Radial Compressor Impeller and Its Effect on the Stage Performance”, ASME Journal of Turbomachinery, December 2014, 136 (12):, 121004 (11 pages); doi:10.1115/1.4028218